Journée Interfaces 2014
25th April 2014 - LPMA, Paris

Disordered Elastic Systems & One-dimensional interfaces

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UNIVERSITÉ DE GENÈVE
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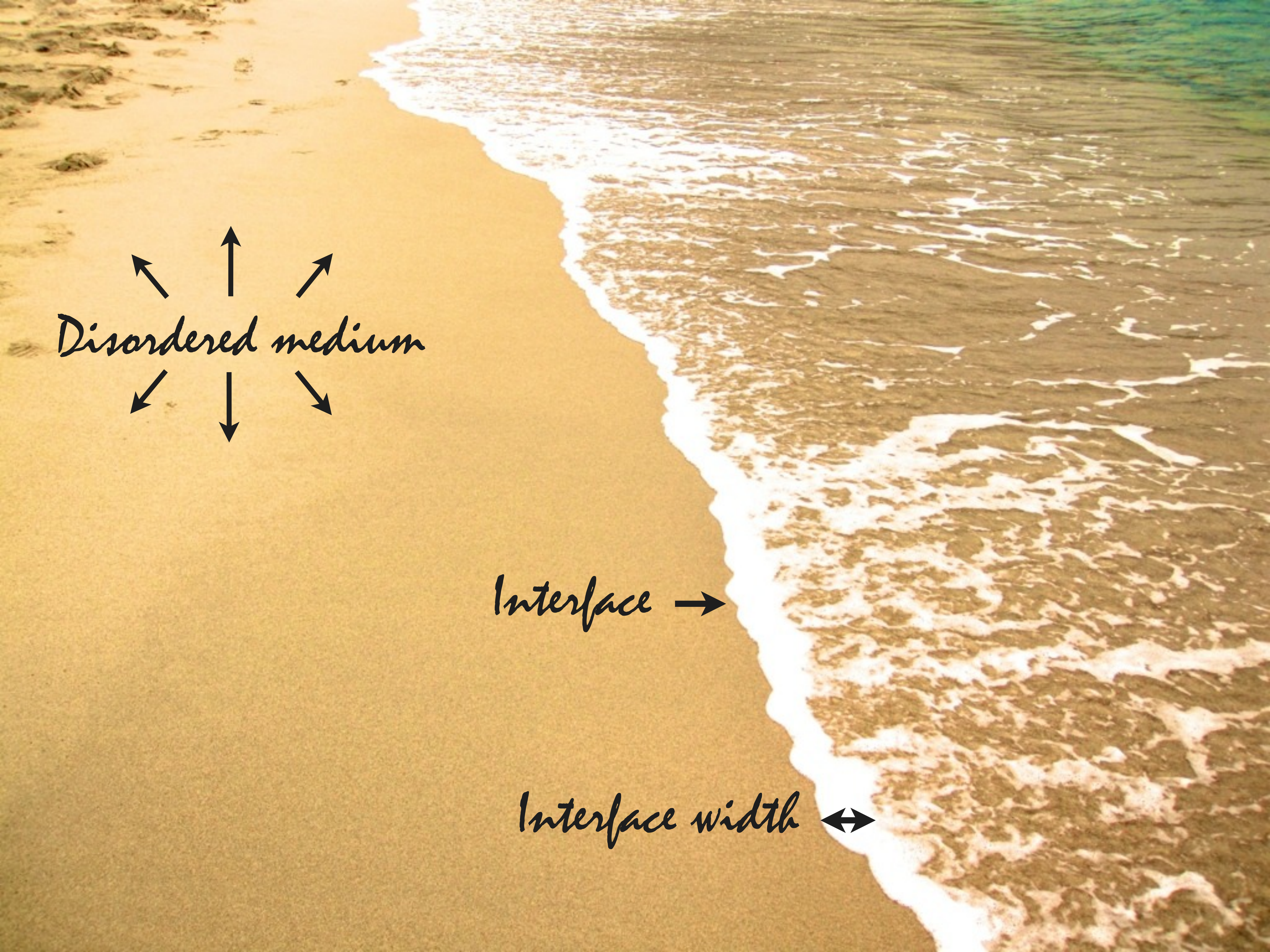

Disordered medium
Interface
Interface width

Interfaces can be found everywhere...

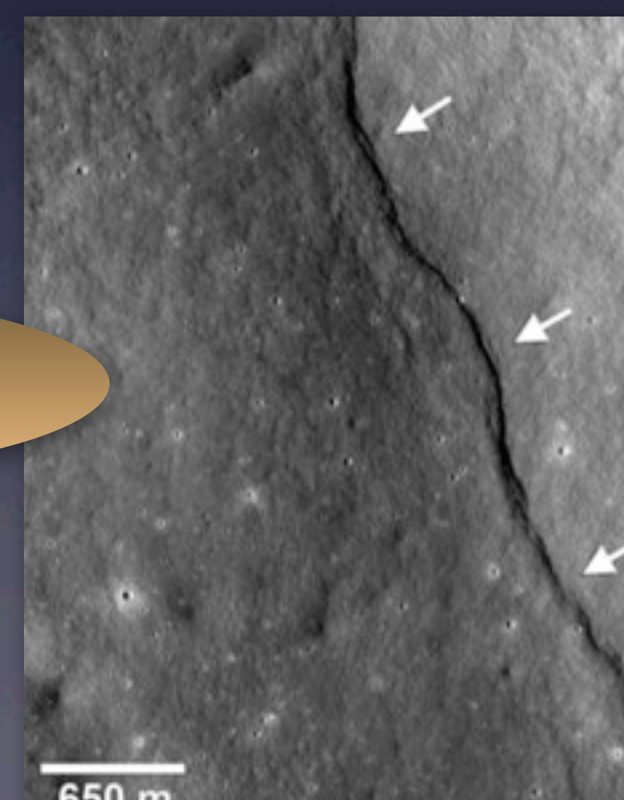
← Scotland coastline

Lengthscale matters!

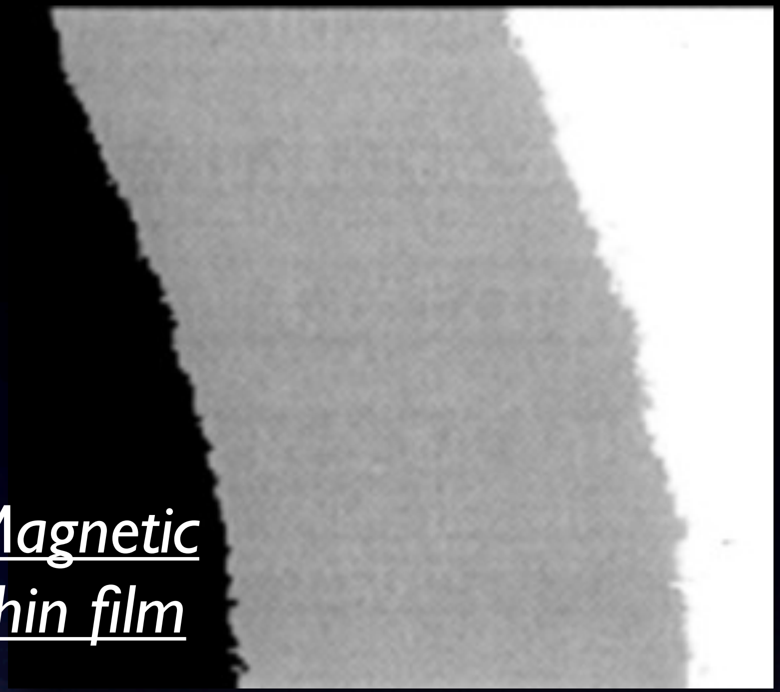
← Crack in a pavement

Scale invariance?

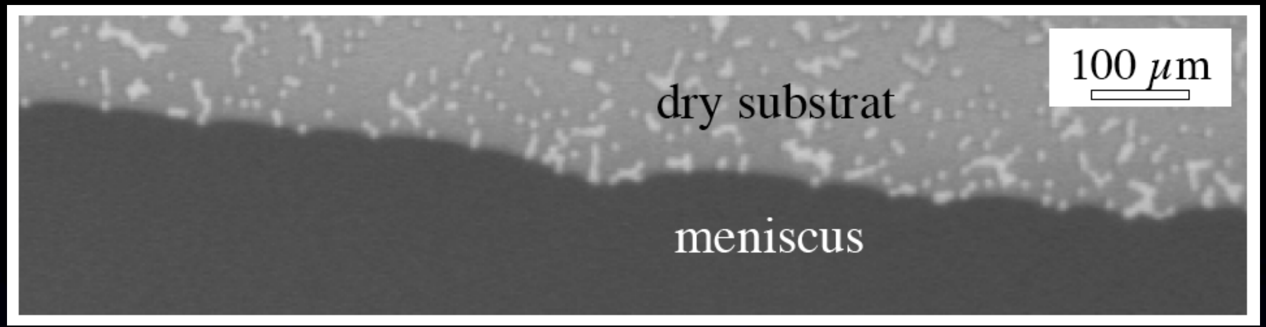
Crack at the Moon's surface →



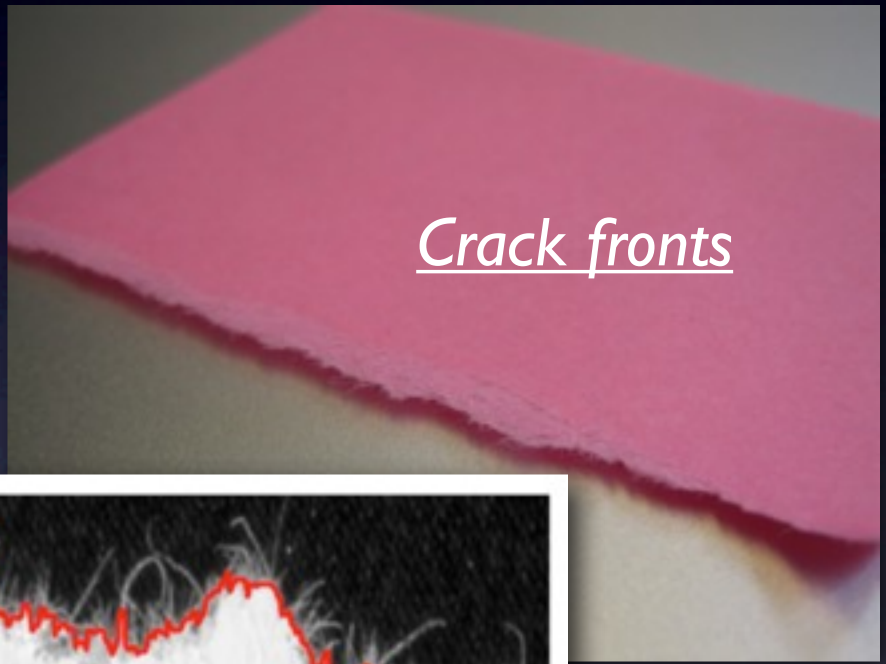
Interfaces can be found everywhere...

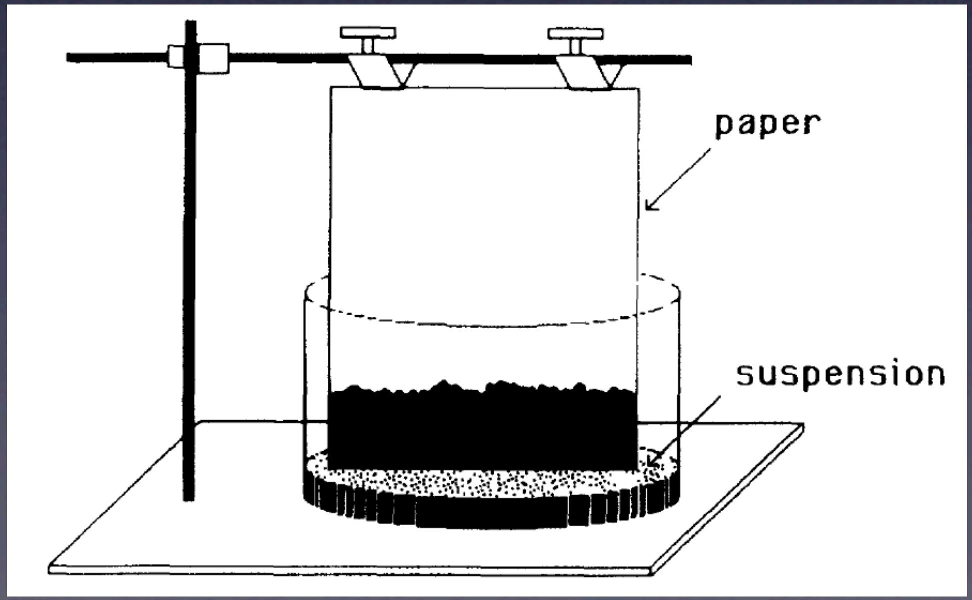


Lemerle *et al.*, *Phys. Rev. Lett.* **80**, 894 (1998).

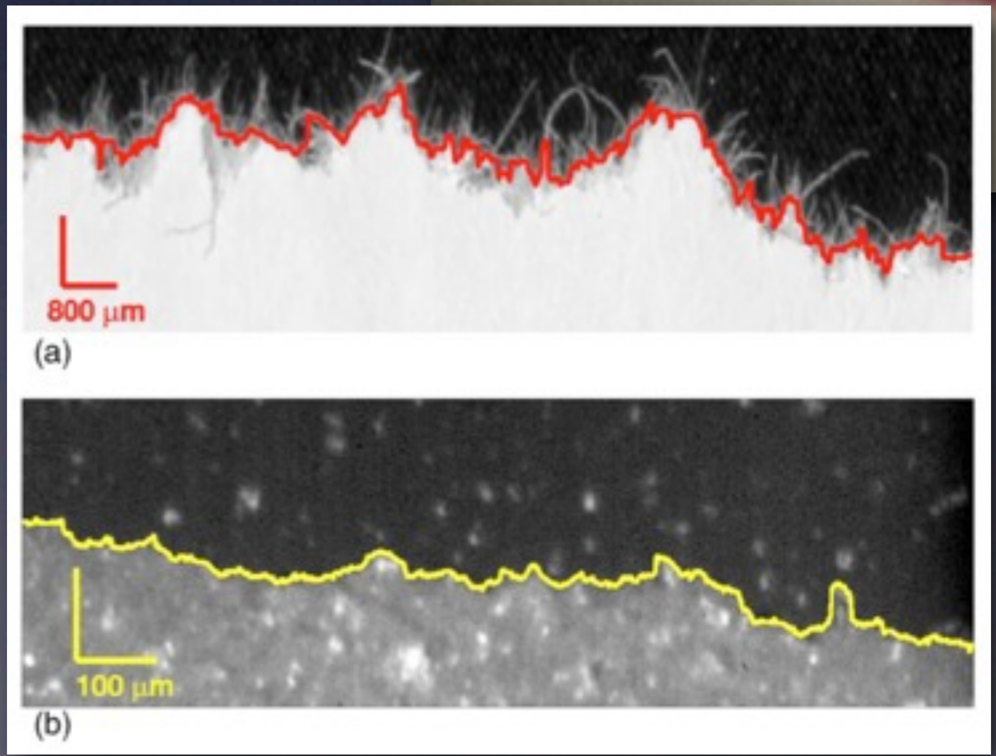


Moulinet *et al.*, *Eur. Phys. J. E* **8**, 437 (2002).





Buldyrev *et al.*, *Phys. Rev. A* **45**, 8313 (1992).

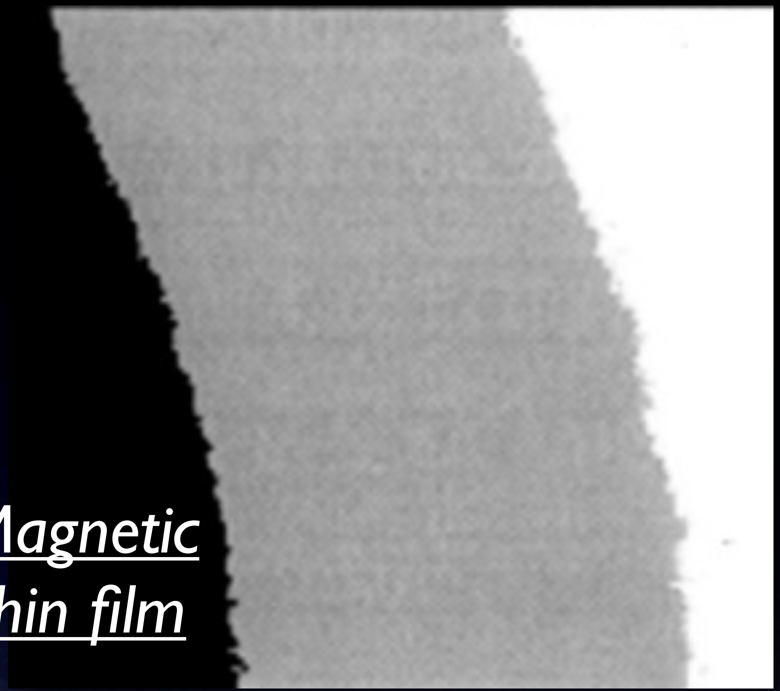


← Paper

← Sandblasted plexiglass

Santucci et al., *Phys. Rev. E* **75**, 016104 (2007).

Interfaces can be found everywhere...



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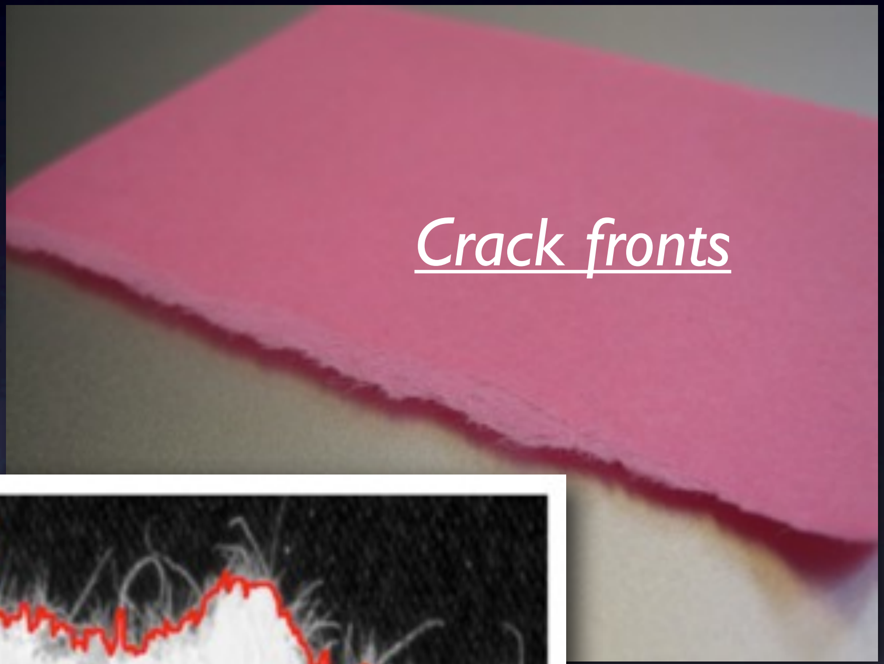
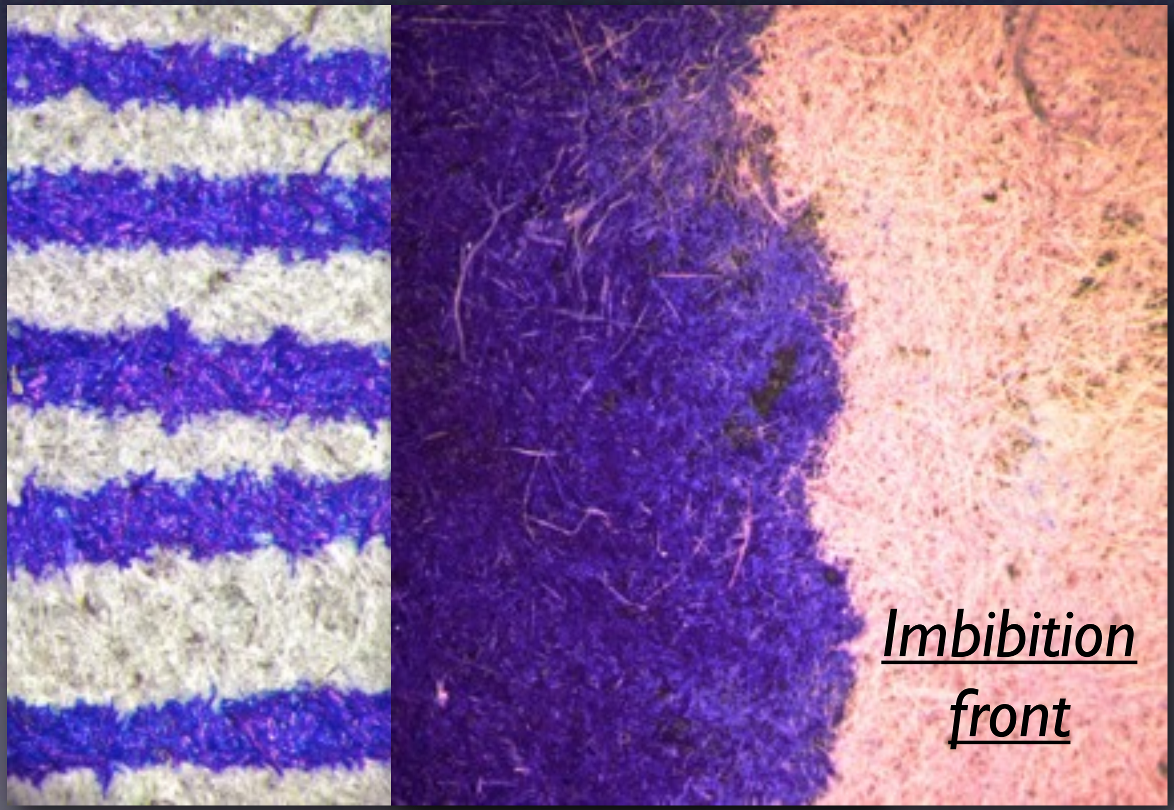
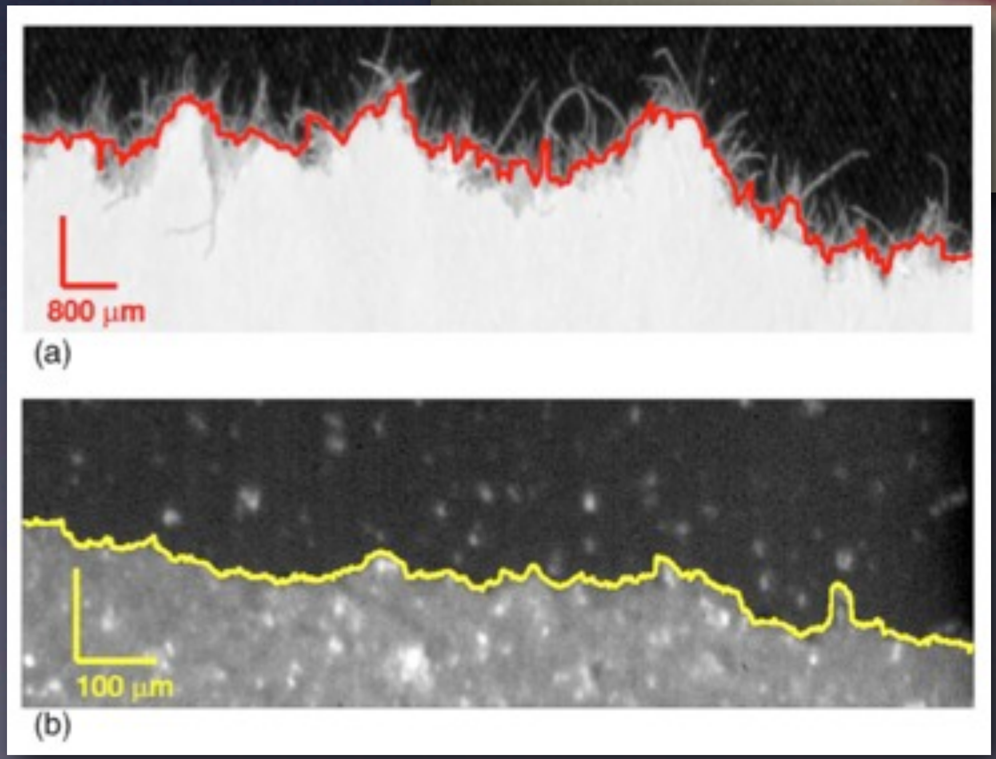






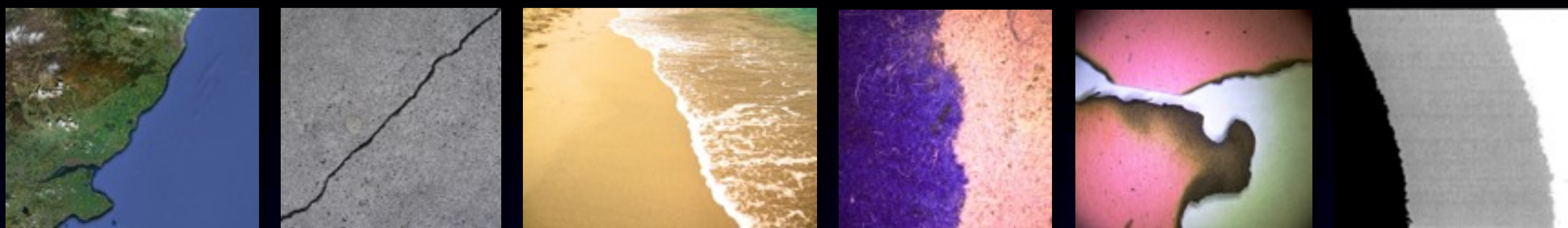
← Paper

← Sandblasted plexiglass

Santucci et al., *Phys. Rev. E* **75**, 016104 (2007).

Interfaces can be found everywhere...

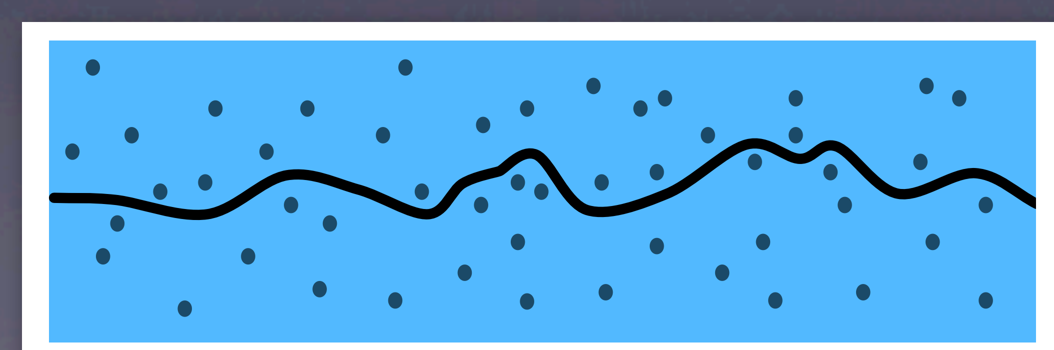
- Ubiquitous in Nature, large variety of lengthscales & microphysics.
  BUT do they share nevertheless common (universal?) features?



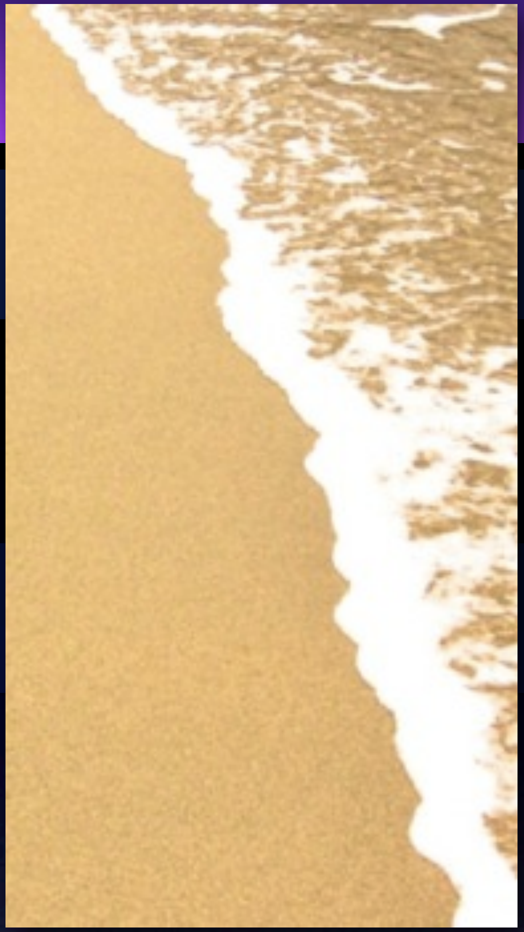
Review: A.-L. Barabási & H. E. Stanley, *Fractal Concepts in Surface Growth*, Cambridge University Press, 1995.

- Increasing complexity starting from a MICROSCOPIC description.
  ⇒ Need of a simpler *MESOSCOPIC* starting point
- Systems supported by an inhomogeneous underlying medium.
  ⇒ Statistical characterization of *DISORDER*
- Effective description depending on the LENGTHSCALE.
  ⇒ Characteristic lengthscales, scale invariance?

How do they look like?
How do they respond when one pulls at them?
Disorder-conditioned features?



Outline



- Introduction
  - Generic framework: Disordered Elastic Systems (DES)
  - Specific issue: role a finite width or disorder correlation length
- Model of a one-dimensional interface
  - Geometrical fluctuations and roughness
  - DES model of a one-dimensional (1D) interface
  - Static 1D interface versus 1+1 Directed Polymer (DP)
- Temperature-dependent fluctuations
  - Disorder free-energy fluctuations
  - Roughness: temperature-induced crossover
- Link to prototypal experiments
  - Ferromagnetic domain walls in ultrathin films
  - Growing interfaces in nematic liquid crystals
- Perspectives
  - 1D Kardar-Parisi-Zhang (KPZ) universality class

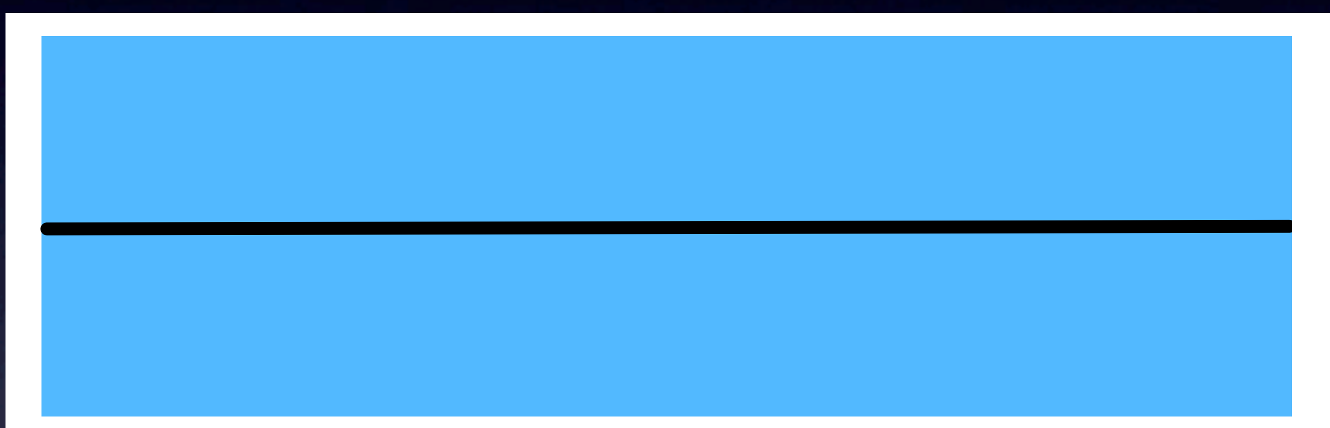
Disordered Elastic Systems (DES)

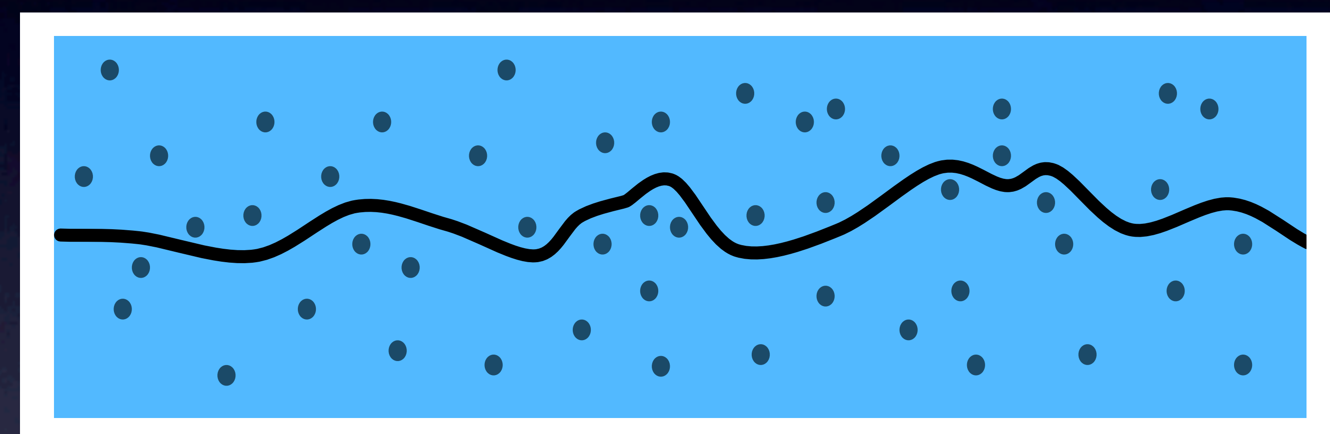
- Competition of three physical ingredients ⇒ METASTABILITY, GLASSY PROPERTIES

ELASTICITY

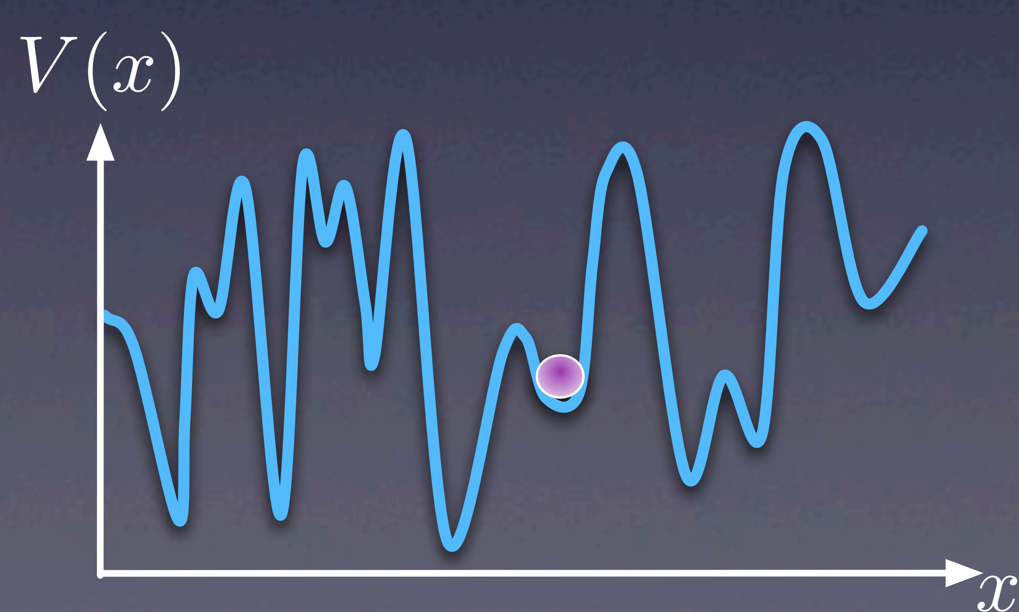
TEMPERATURE

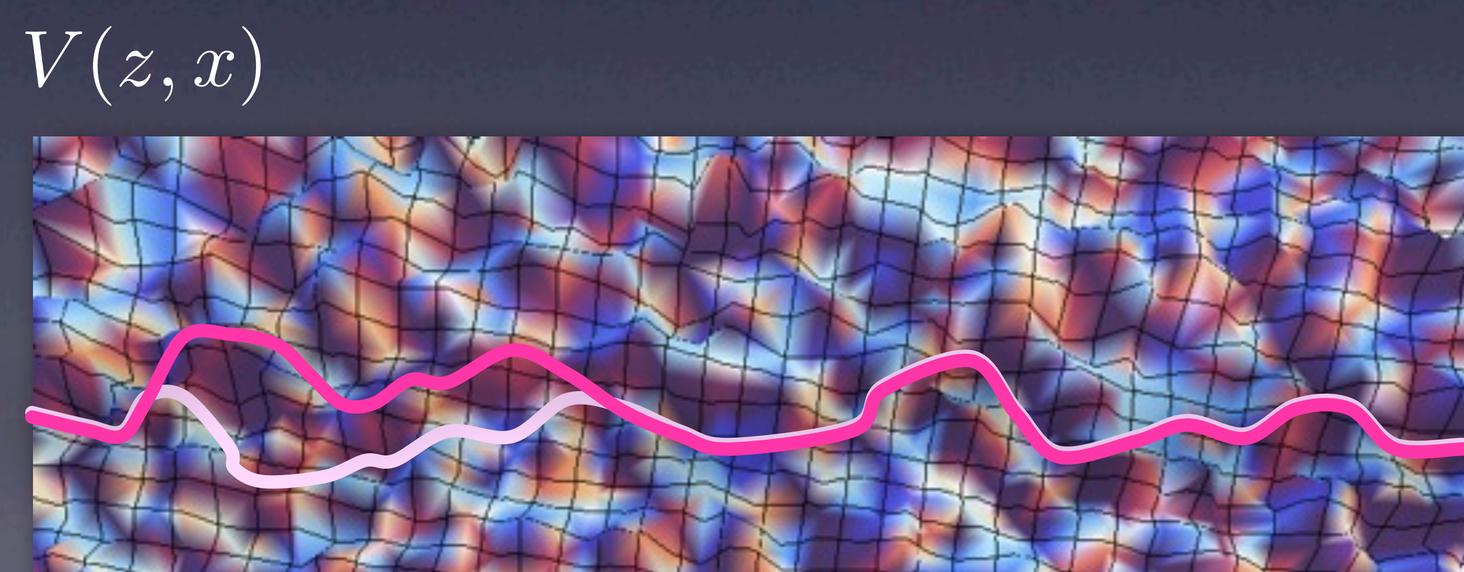
DISORDER





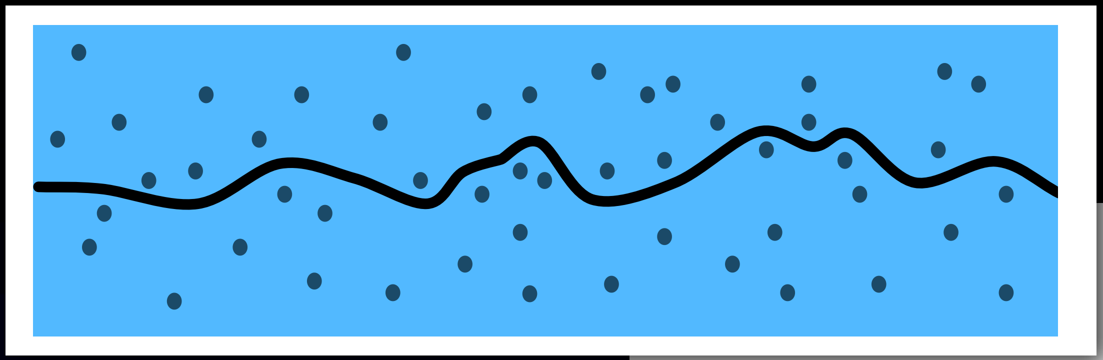
- Exploration of disordered energy landscapes





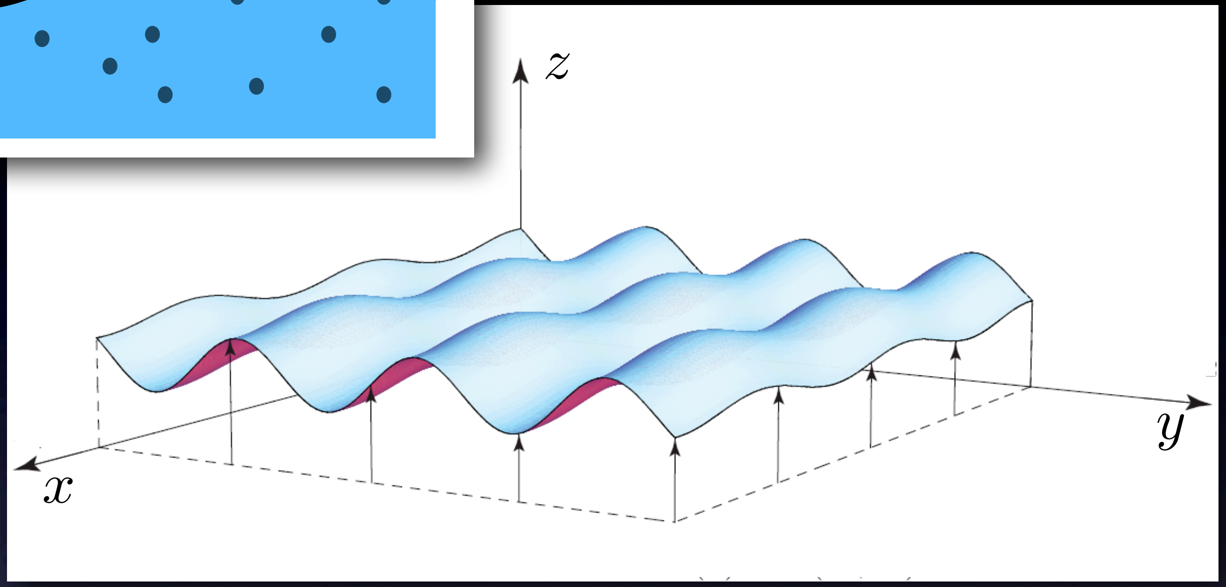
Disordered Elastic Systems (DES): a recipe

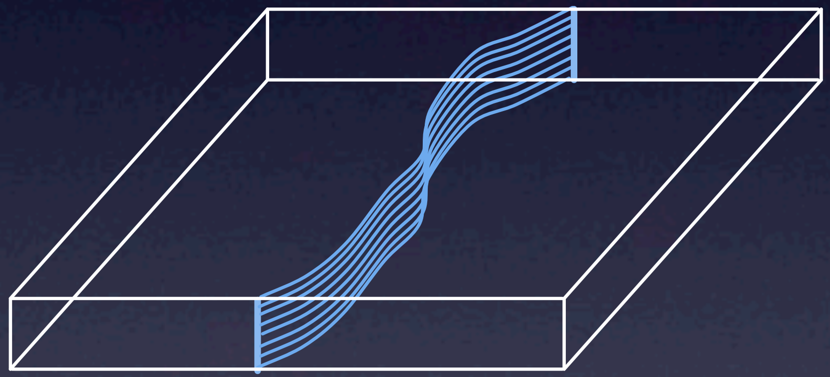
- **Dimensionality**



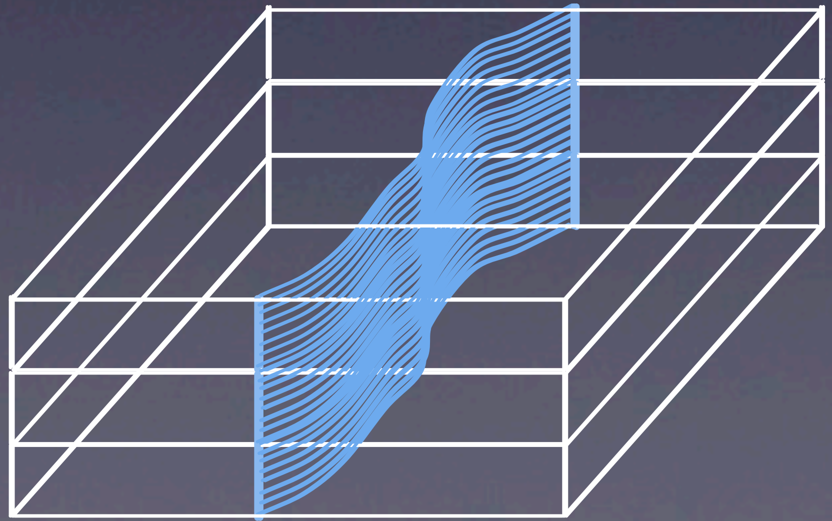
← 1D interface

2D interface →

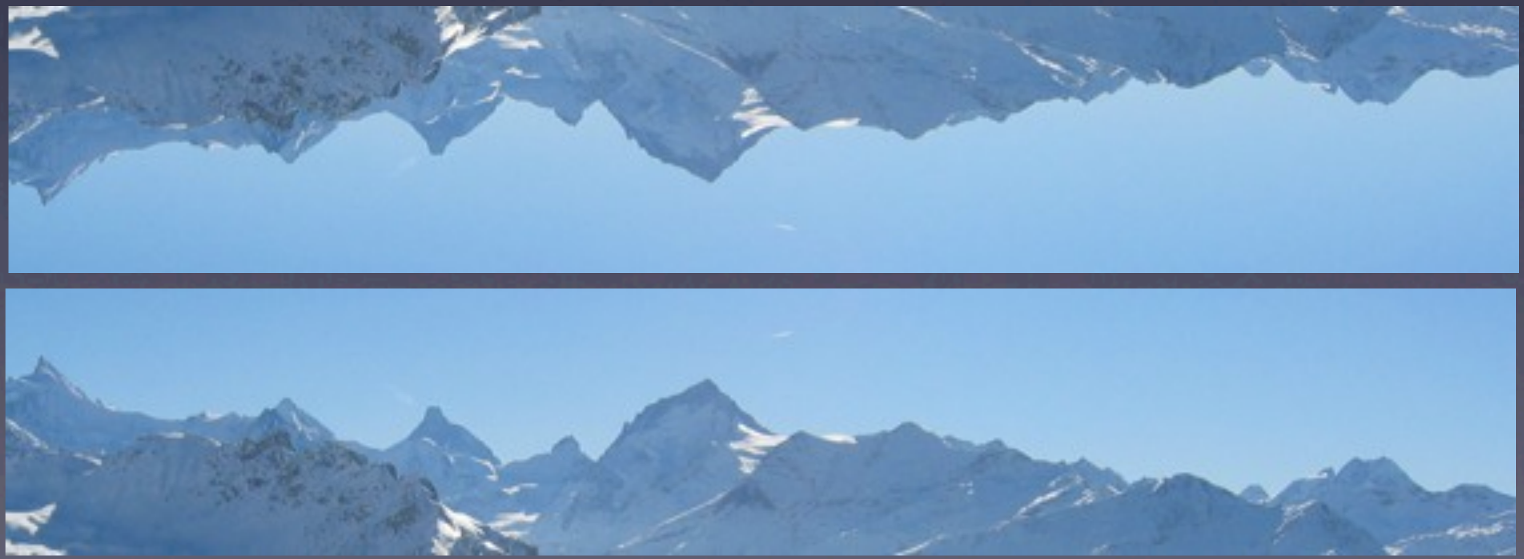




Dimensional crossover?



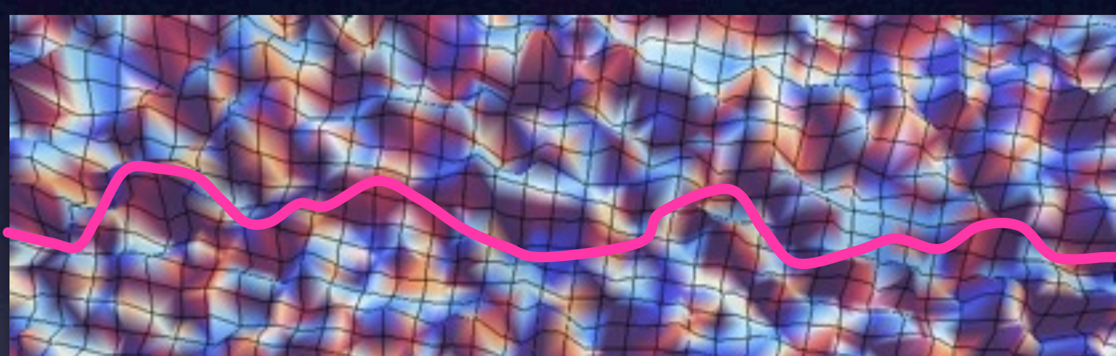
Effective 1D interface? NO!

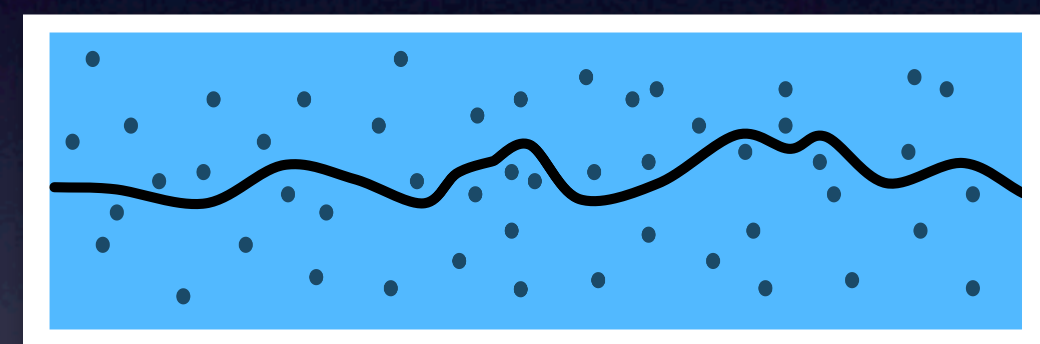


Disordered Elastic Systems (DES): a recipe

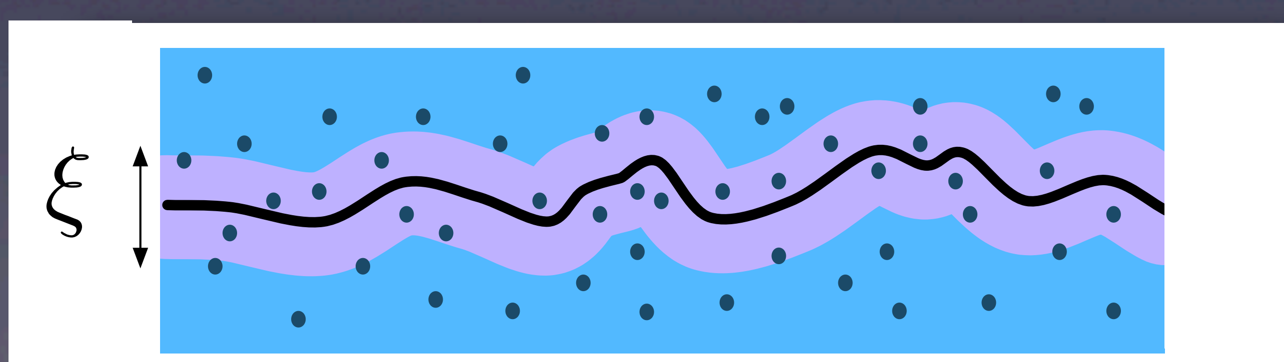
- **Dimensionality**
- **Elasticity:** Short-range *versus* long-range, e.g. $\mathcal{H}_{\mathrm{el}} \propto \text{system size}$
- **Disorder:**
  - Quenched *versus* annealed disorder
  - 'Random-bond' *versus* 'random-field'
  - Collective weak pinning *versus* strong individual pinning centers

$$\mathcal{H}_{\mathrm{DES}} = \mathcal{H}_{\mathrm{el}} + \mathcal{H}_{\mathrm{dis}}$$

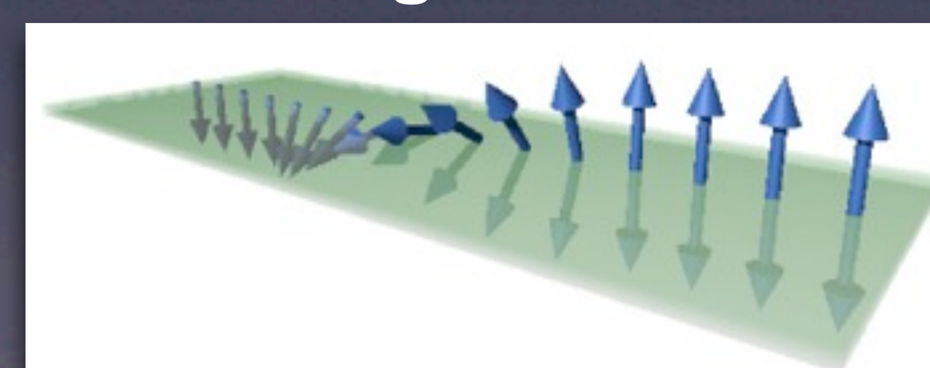




- **No bubbles nor overhangs**
- **Finite width / Disorder correlation**



- **Internal degree of freedom?**



E. Agoritsas, V. Lecomte & T. Giamarchi, *Physica B* **407**, 1725 (2012).

Observables for probing disordered systems

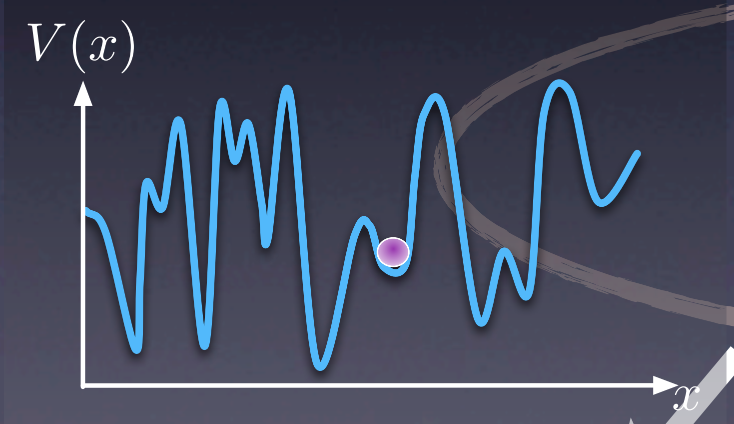
How do they look like?

How do they respond when one pulls at them?

Disorder-conditioned features?

- Dimensionality
- Elasticity
- Disorder

- No bubbles/overhangs
- Internal structure
- At (non-)equilibrium



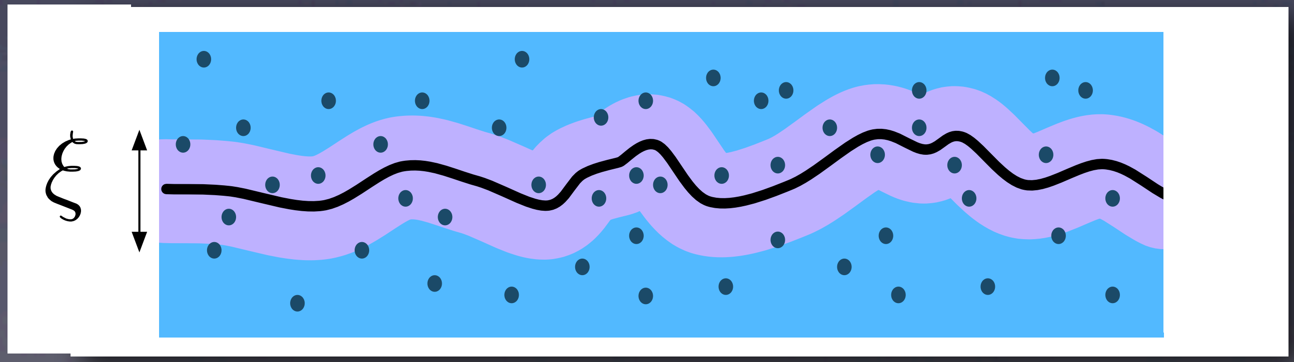
Probe of disorder-conditioned features
in STATICS/DYNAMICS

Geometrical fluctuations
& roughness

Steady-state velocity
under an external force

Main issue

What is the imprint of a finite microscopic width and/or disorder correlation length ξ on the 1D interface fluctuations and properties?



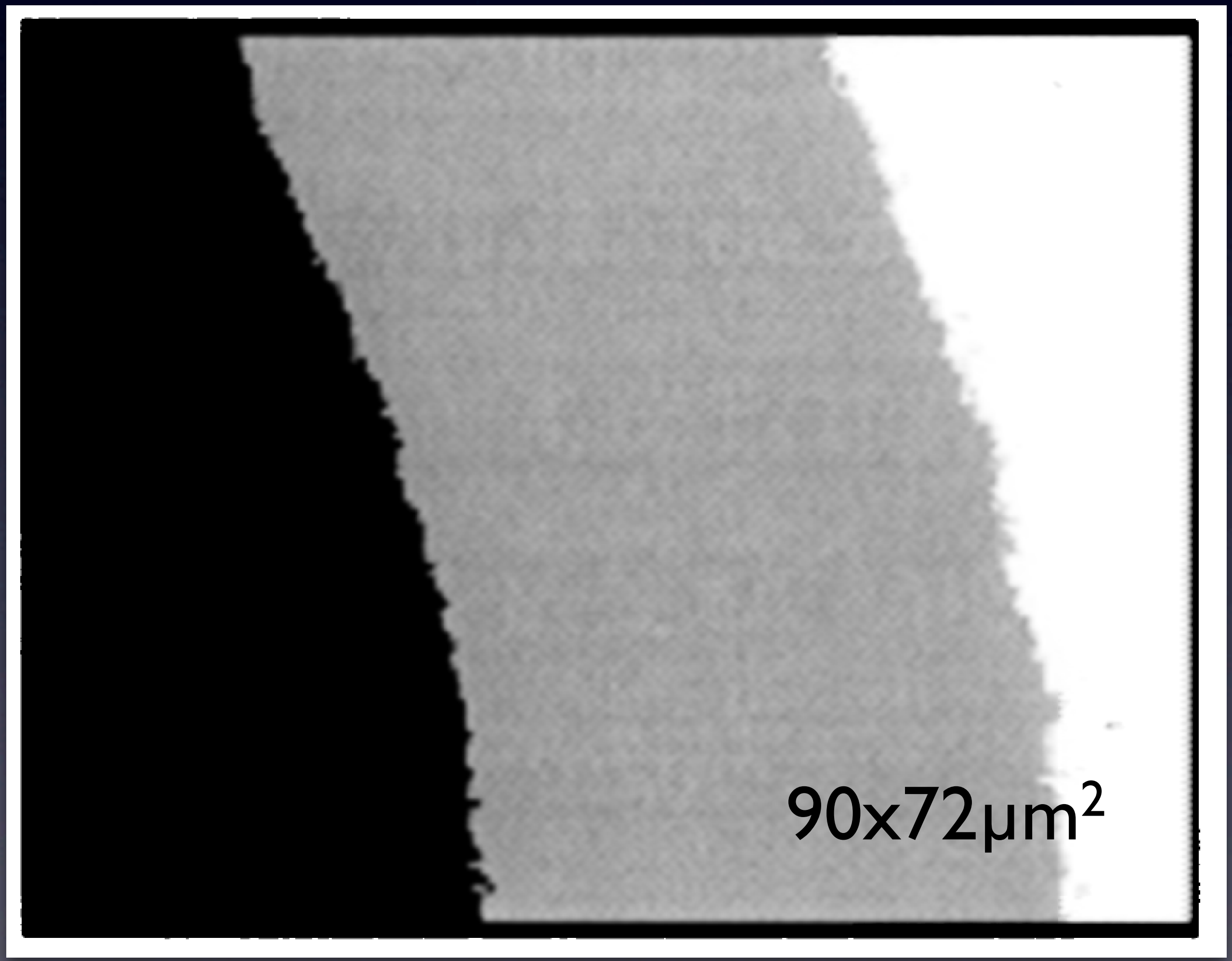
Main issue: finite width or disorder correlation length $\xi > 0$

- Two examples of experimental realizations of interfaces:

Ferromagnetic domain wall $(\xi \sim 50\text{nm})$

RESOLUTION: 1µm

Ultrathin film of Pt/Co/Pt (a few atomic layers)

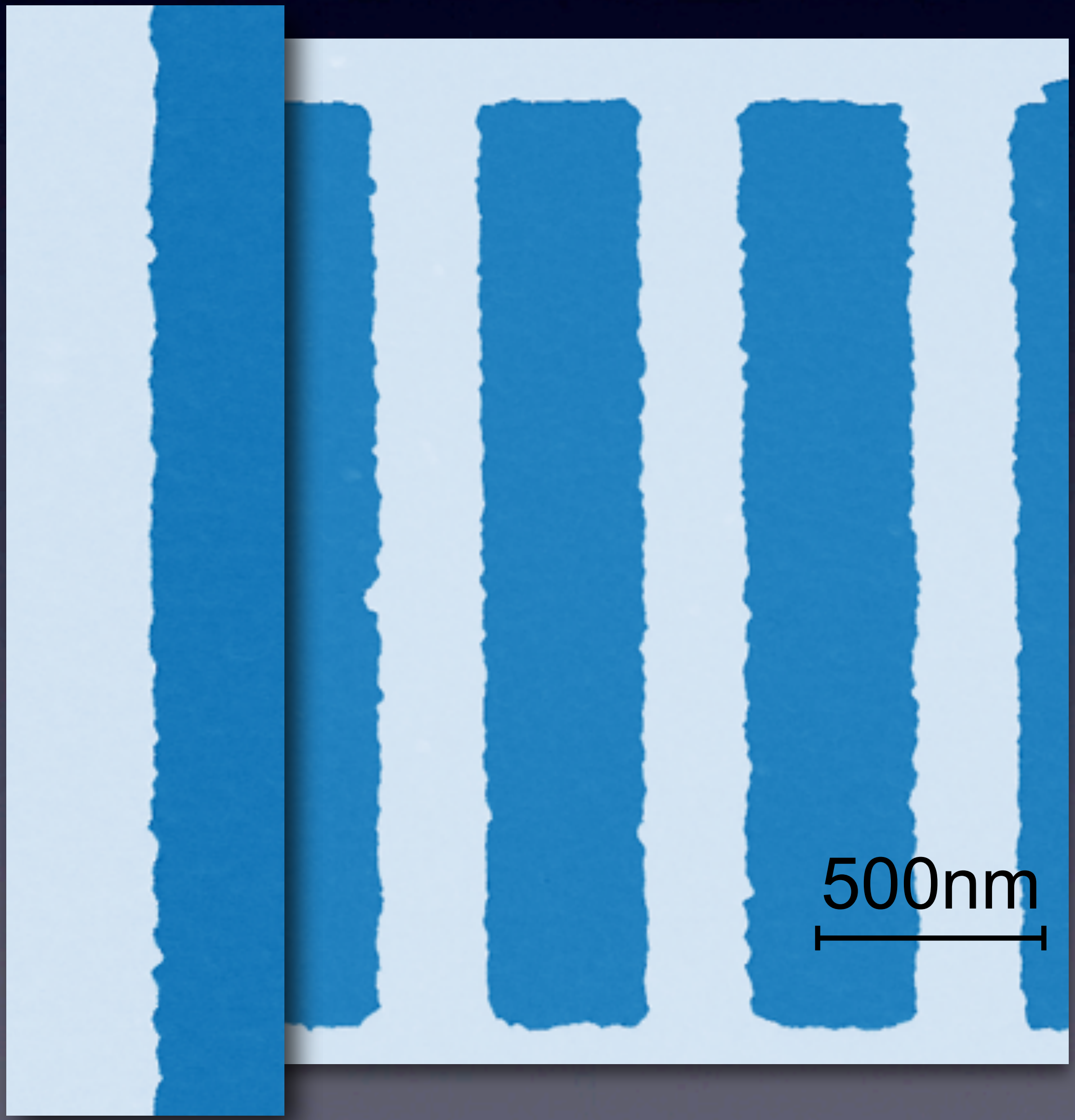


S. Lemerle, J. Ferré, C. Chappert, V. Mathet, T. Giamarchi, & P. Le Doussal, *Phys. Rev. Lett.* **80**, 849 (1998).

Ferroelectric domain wall $(\xi \sim 1\text{nm})$

RESOLUTION: 5nm

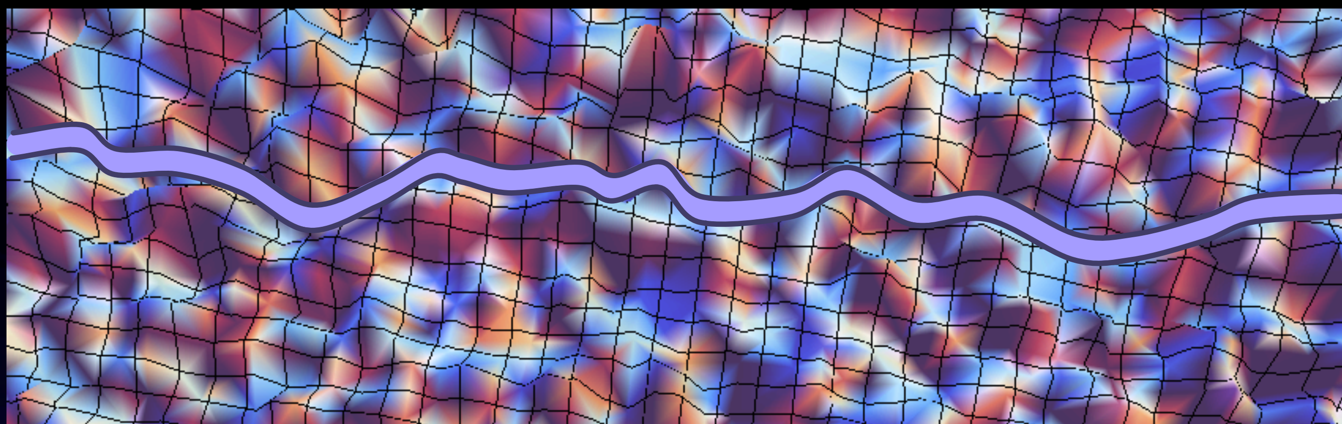
$PbZr_{0.2}Ti_{0.8}O_3$ 70nm / $SrRuO_3$ 30nm (electrode) / $SrTiO_3$ (substrate)



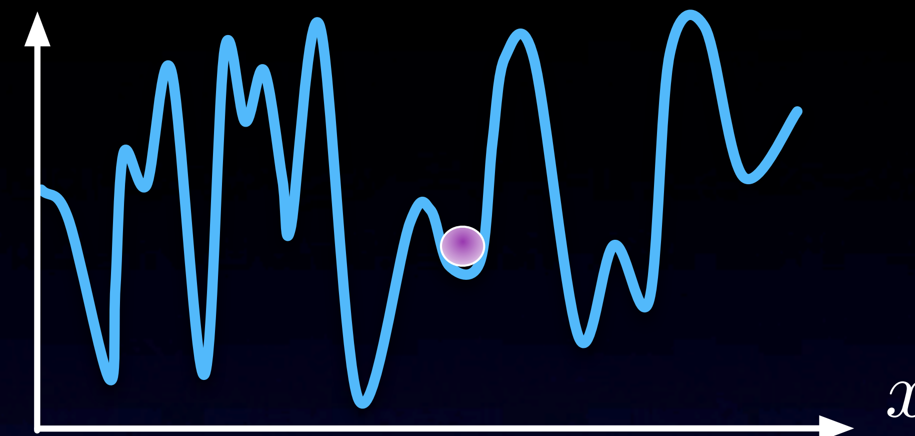
Courtesy of J. Guyonnet & Prof. P. Paruch.

Main result: low-temperature regime at $\xi > 0$

Random potential: $V(z, x)$



$V(x)$

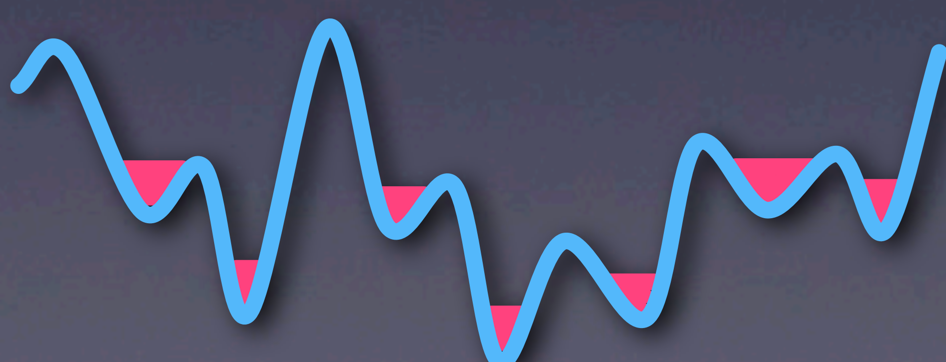


Interplay between

- Thermal fluctuations $T > 0$
- Width and/or disorder correlation length $\xi > 0$

$\Longrightarrow$

$$\xi_{\mathrm{thermal}}(T) \lessgtr \xi$$
$$\Updownarrow$$
$$T \lessgtr T_c(\xi)$$



Low temperature $\xi_{\mathrm{th}}(T) \ll \xi$

High temperature $\xi_{\mathrm{th}}(T) \gg \xi$

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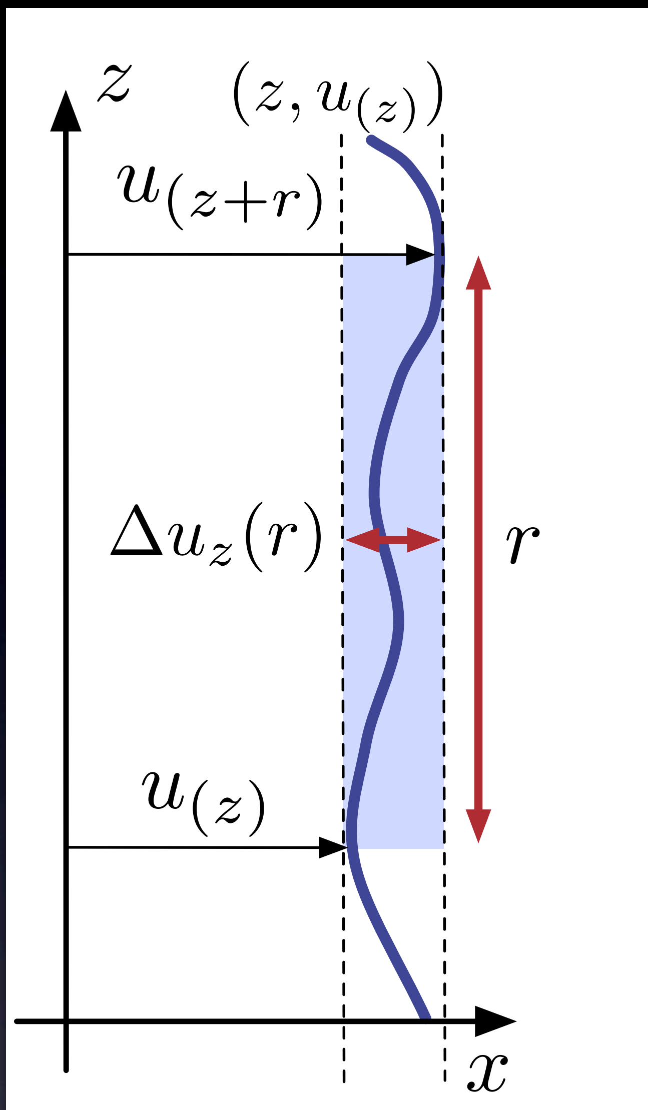
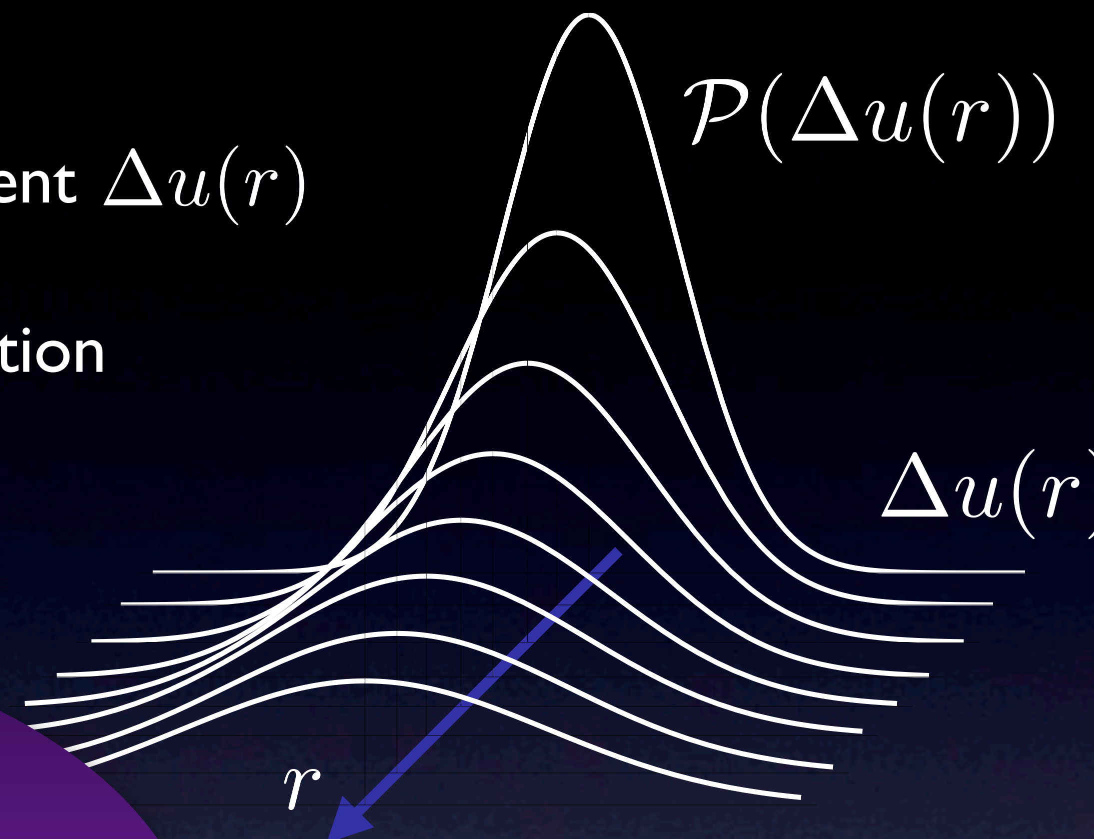
Geometrical fluctuations & roughness



- Lengthscale r
  - Relative displacement $\Delta u(r)$

- Probability distribution function

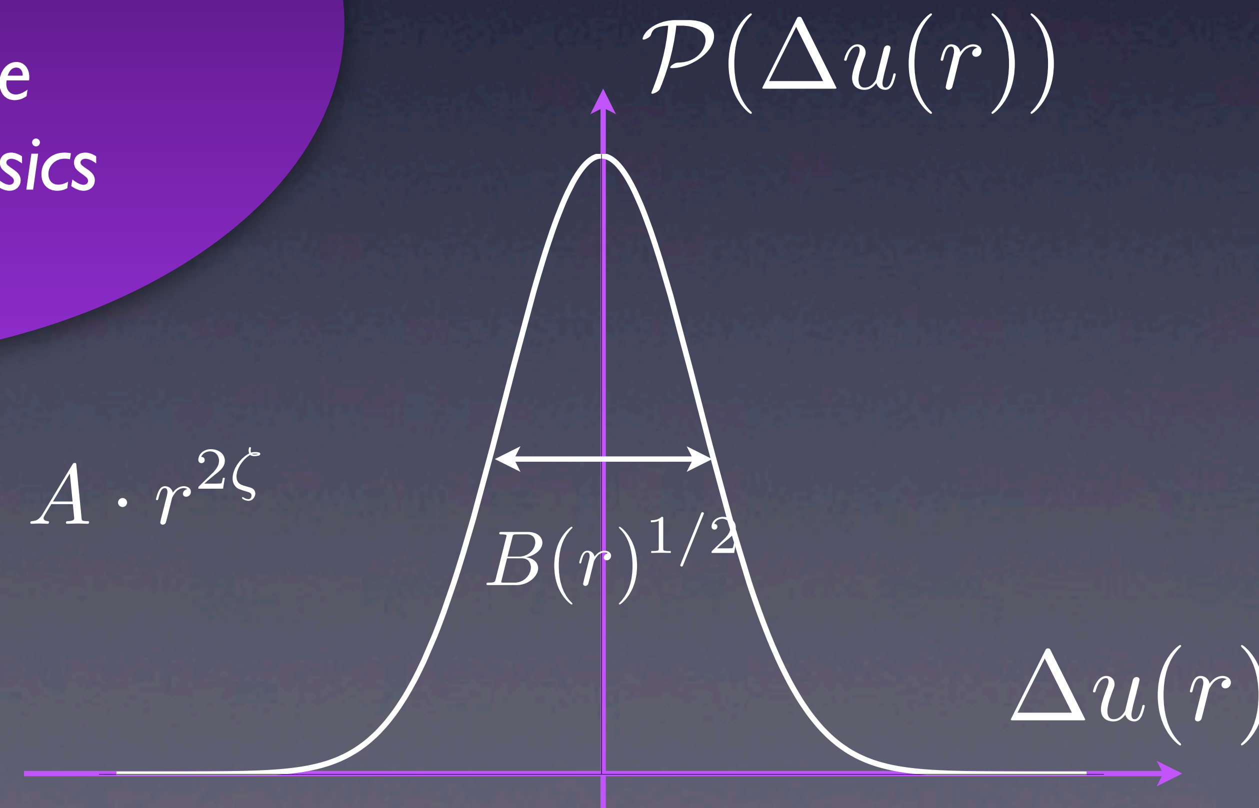
$$\mathcal{P}(\Delta u(r))$$



Roughness exponent ζ

=

Signature of the predominant physics

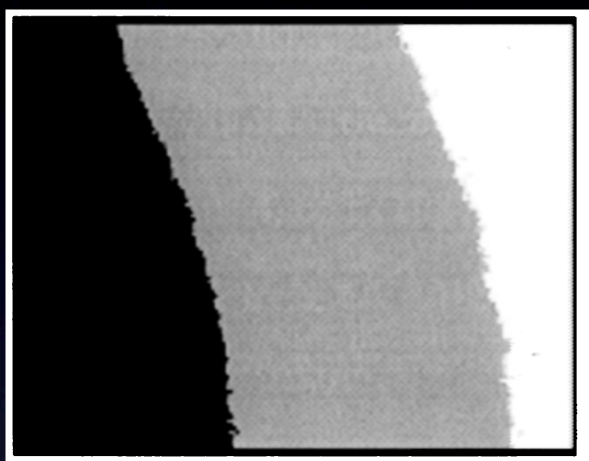


- Roughness function $B(r) = \overline{\langle \Delta u(r)^2 \rangle} \sim A \cdot r^{2\zeta}$

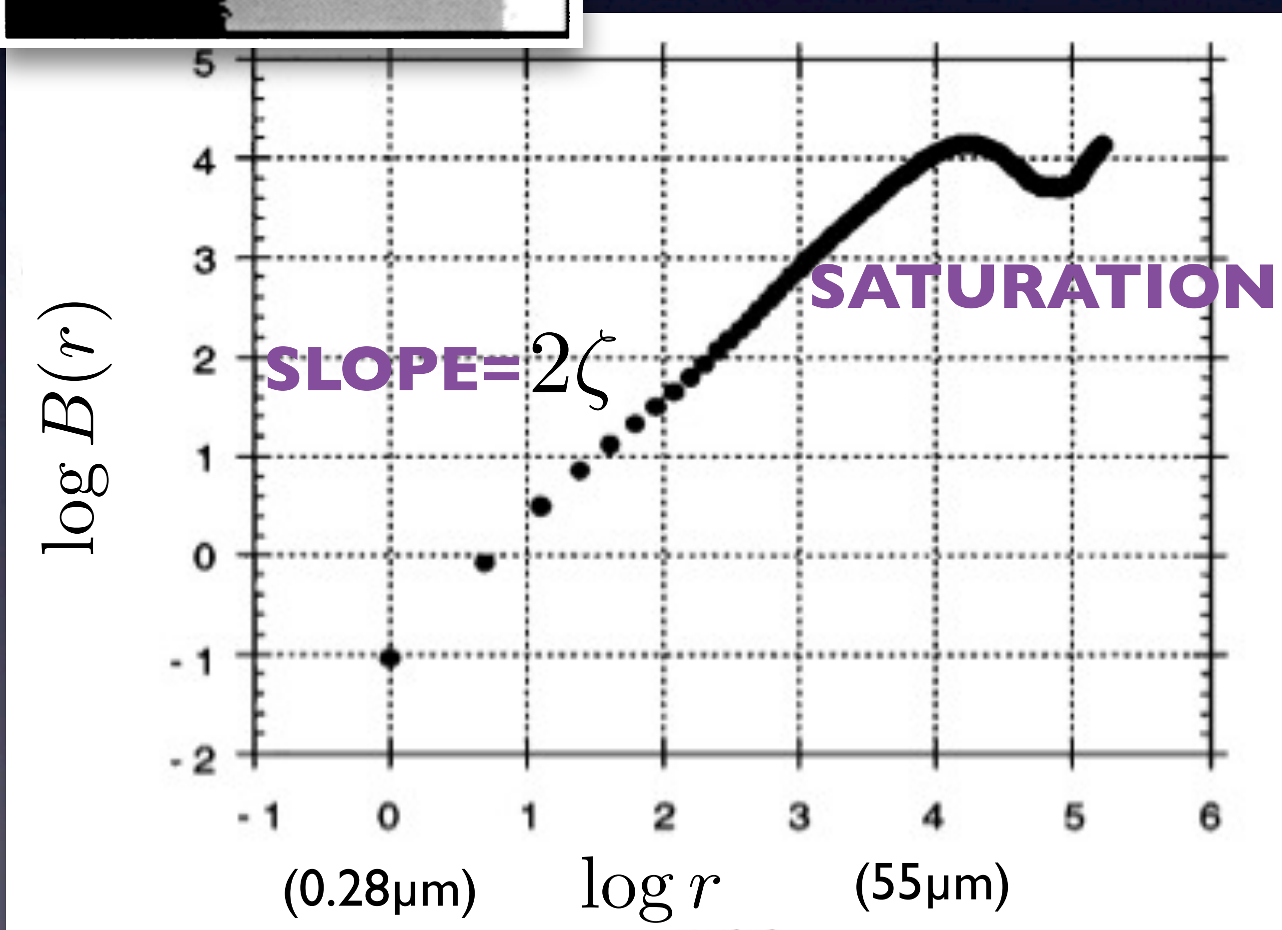
- Roughness exponent: $\begin{cases} \zeta_{\mathrm{thermal}} = 1/2 \\ \zeta_{\mathrm{KPZ}} = 2/3 \end{cases}$

Geometrical fluctuations & roughness: experimental examples

Domain walls in ultrathin Pt/Co/Pt ferromagnetic films

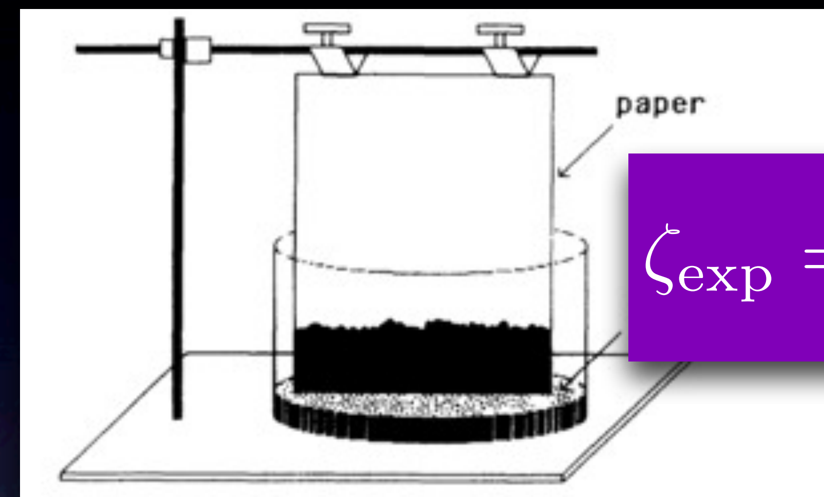


$$\zeta_{\rm exp} = 0.69 \pm 0.07$$

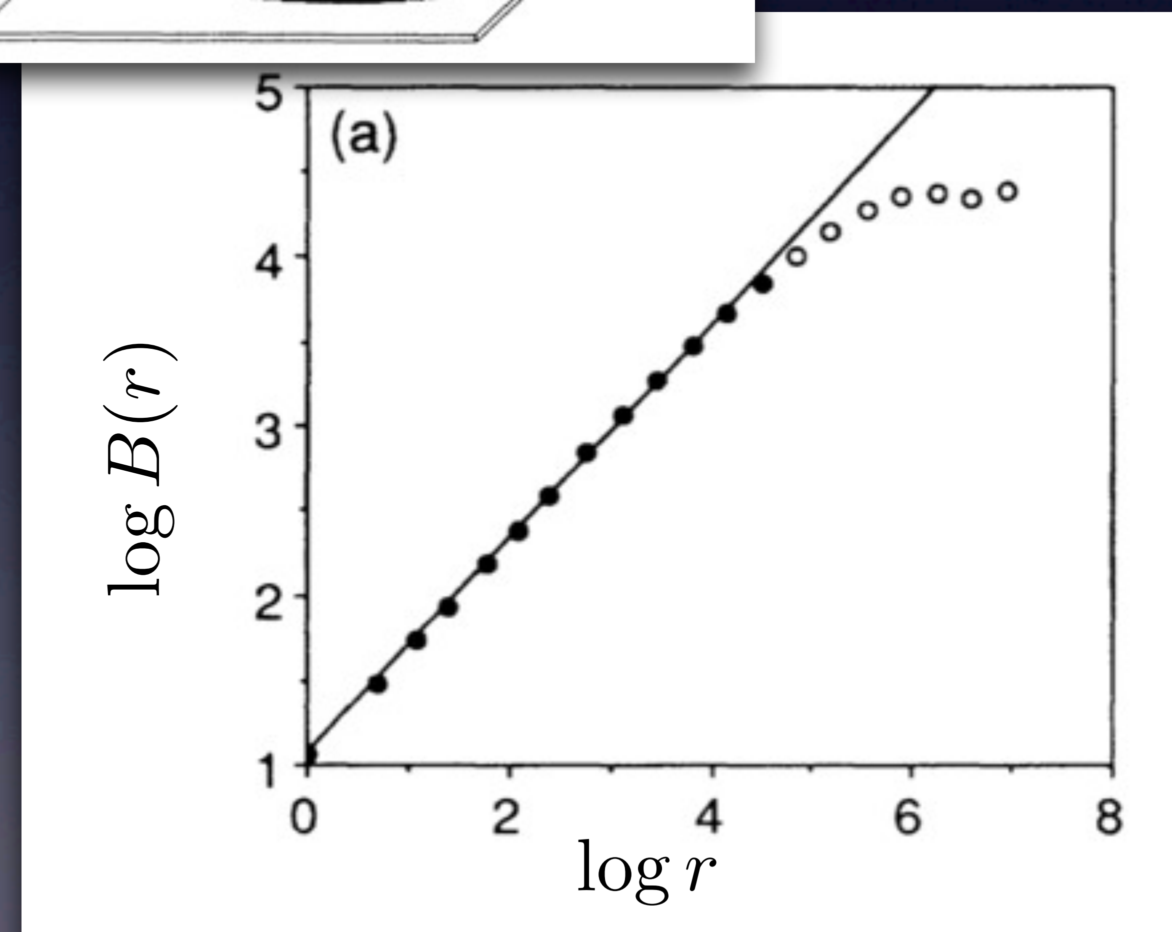


S. Lemerle et al., *Phys. Rev. Lett.* **80**, 849 (1998).

Fluid invasion in a porous medium



$$\zeta_{\rm exp} = 0.63 \pm 0.04$$



Buldyrev et al., *Phys. Rev. A* **45**, 8313 (1992).

*Model of a **thick** 1D interface & 1+1 Directed Polymer (DP)*

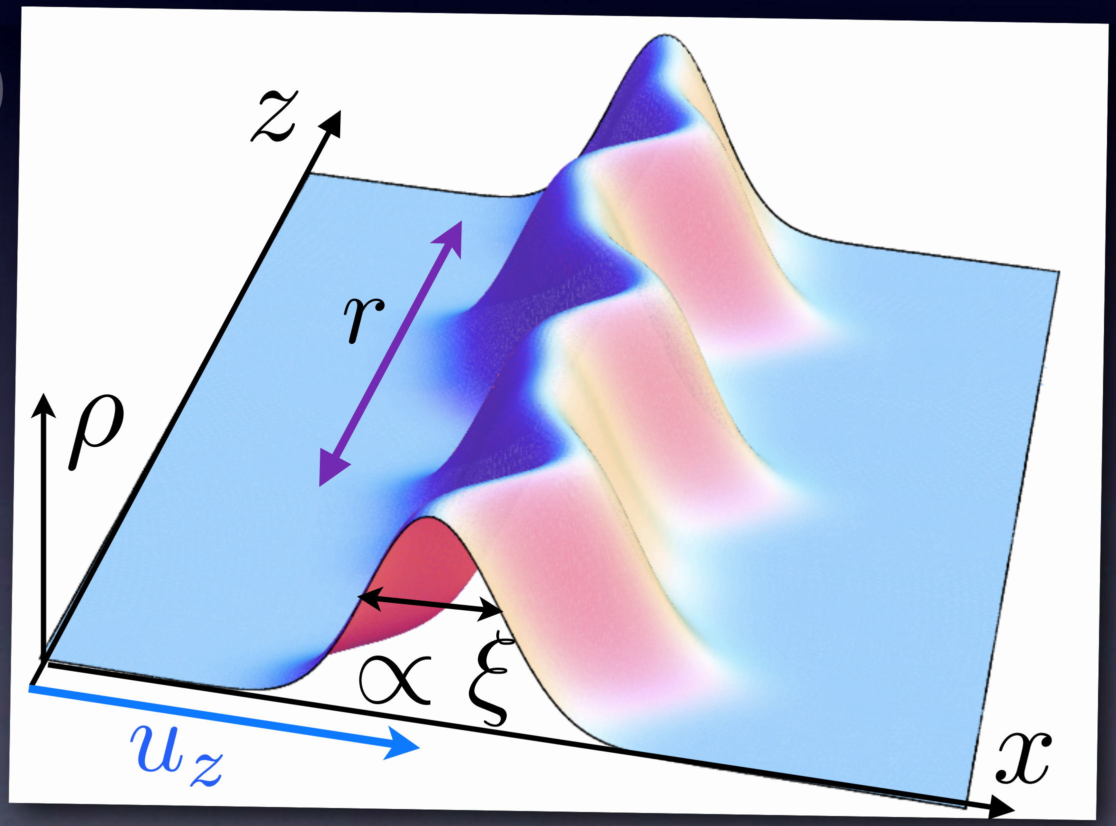
- Short-range elasticity & Elastic limit / Quenched random-bond weak disorder

Hamiltonian: $$\mathcal{H}\left[u,\widetilde{V}\right] = \int_{\mathbb{R}} dz \cdot \left[\frac{c}{2}\left(\nabla_z u_z\right)^2 + \underbrace{\int_{\mathbb{R}} dx \cdot \rho_\xi(x-u_z)\widetilde{V}(z,x)}_{V(z,u_z)} \right]$$

- Density $\rho_\xi(x-u_z)$ & random potential $\widetilde{V}(z,u_z)$

$$\overline{\widetilde{V}(z,x)} = 0$$

$$\overline{\widetilde{V}(z,x)\widetilde{V}(z',x')} = D\cdot\delta_{(z-z')}\delta_{(x-x')}$$



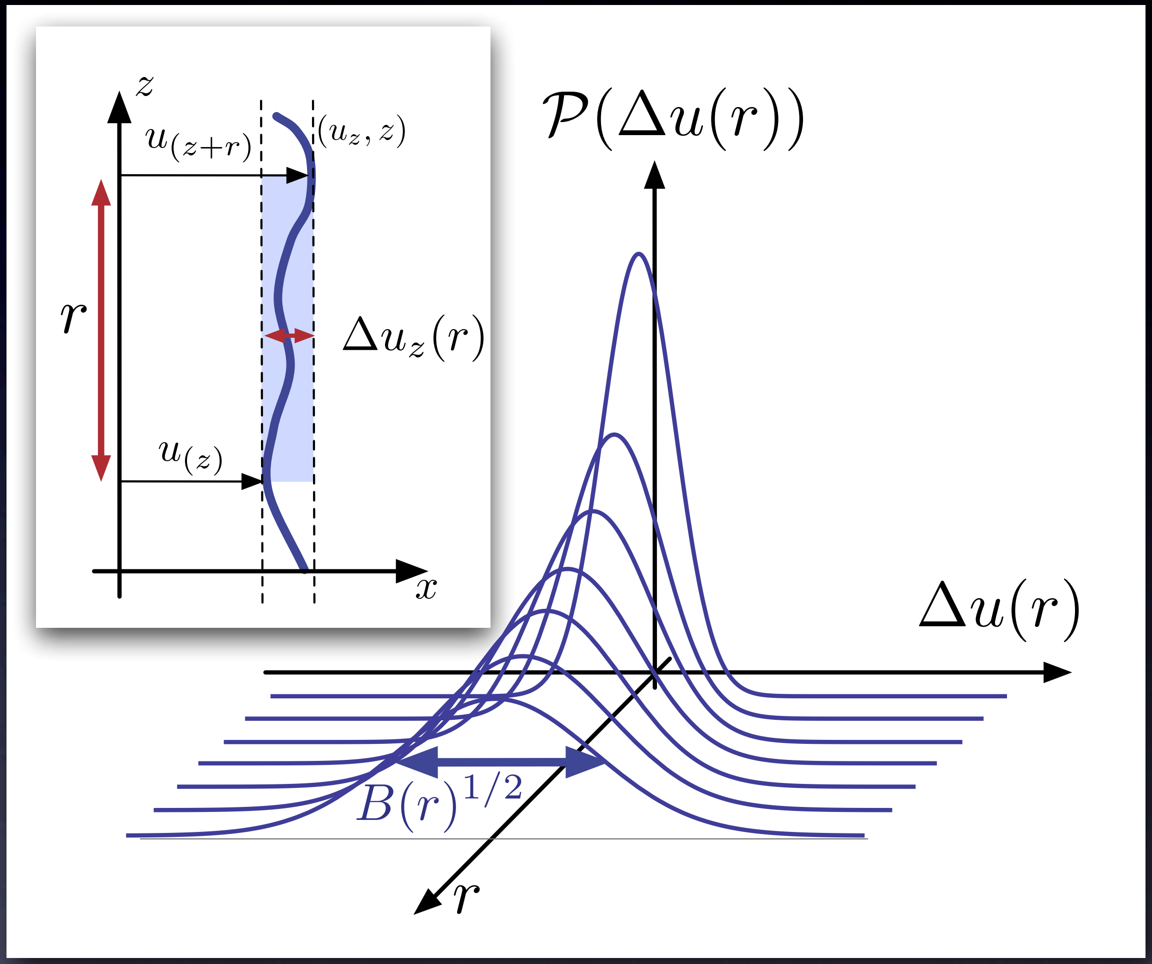
- Alternative: *correlated* effective potential $V(z,u_z)$

$$\overline{V(z,x)V(z',x')} = D\cdot\delta_{(z-z')}R_\xi(x-x')$$

Elastic constant c / Width ξ / Disorder strength D / Temperature T

Issues regarding the roughness at $\xi > 0$

- How many roughness regimes ?
  Characteristic *crossover lengthscales* ?
- Universal roughness amplitude ?
  $$B(r, c, D, T, \xi) \sim A_{(c,D,T,\xi)} \cdot r^{2\zeta}$$
- Imprint of the disorder correlator $R_\xi(x)$?



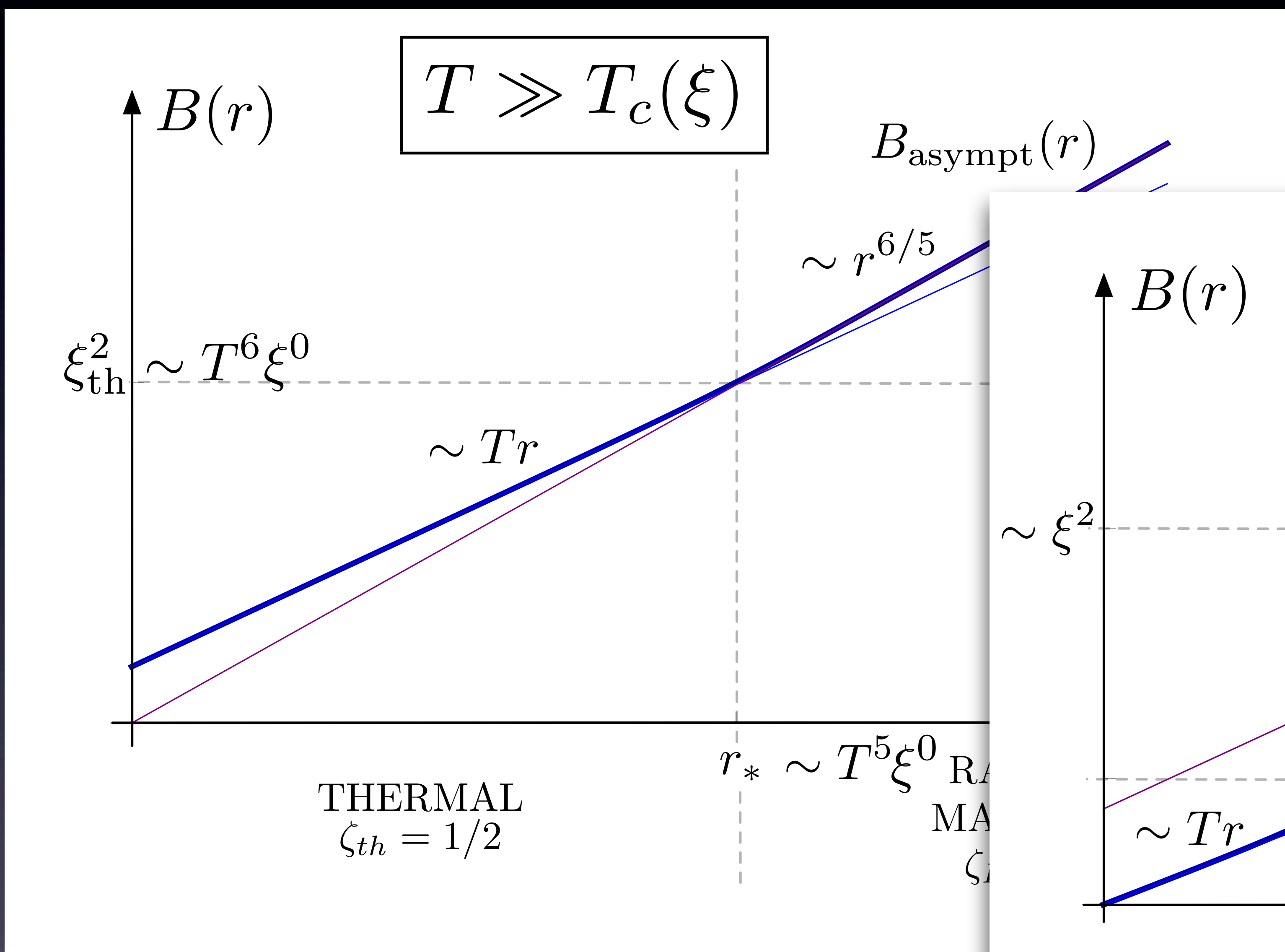
Gaussian-Variational-Method (GVM) on the Hamiltonian

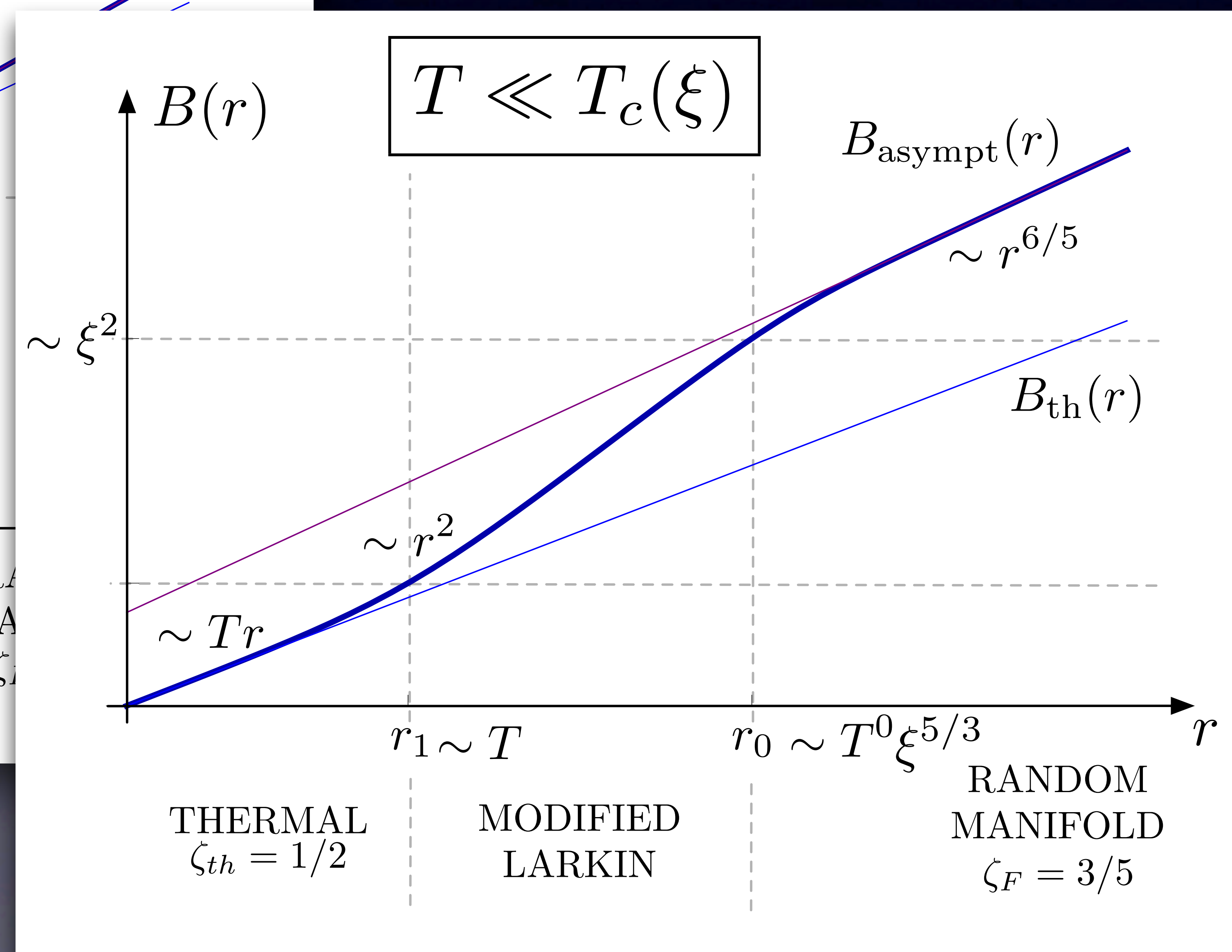
- Two regimes in temperature

$$T_c(\xi) = (\xi c D)^{1/3}$$

- Artefact: wrong asymptotic exponent!

$$B(r) \sim r^{2\zeta} \quad \& \quad \zeta_{\mathrm{RM}}^{\mathrm{exact}} = 2/3$$





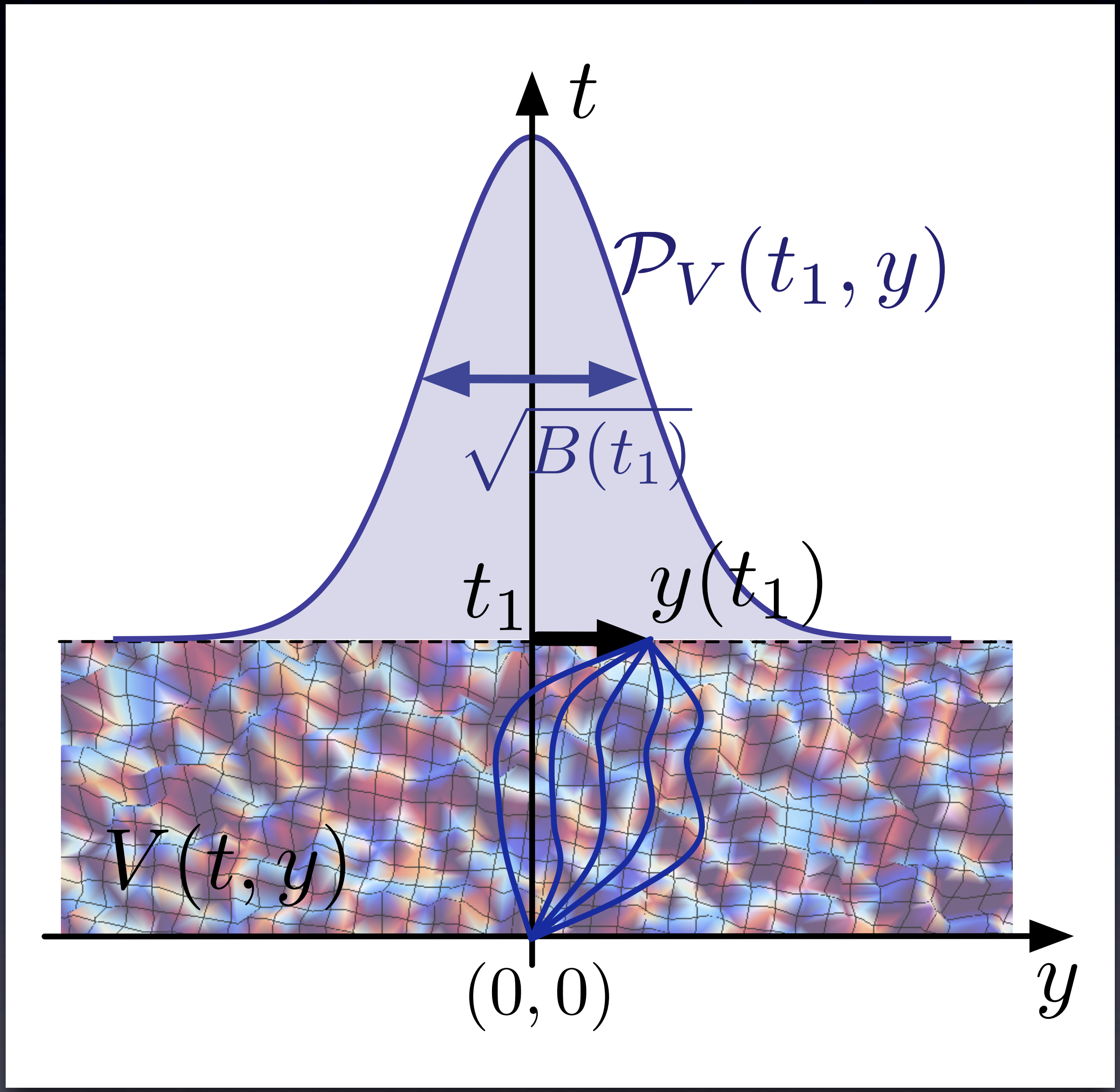
(logarithmic scale)

E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. B* **82**, 184207 (2010).

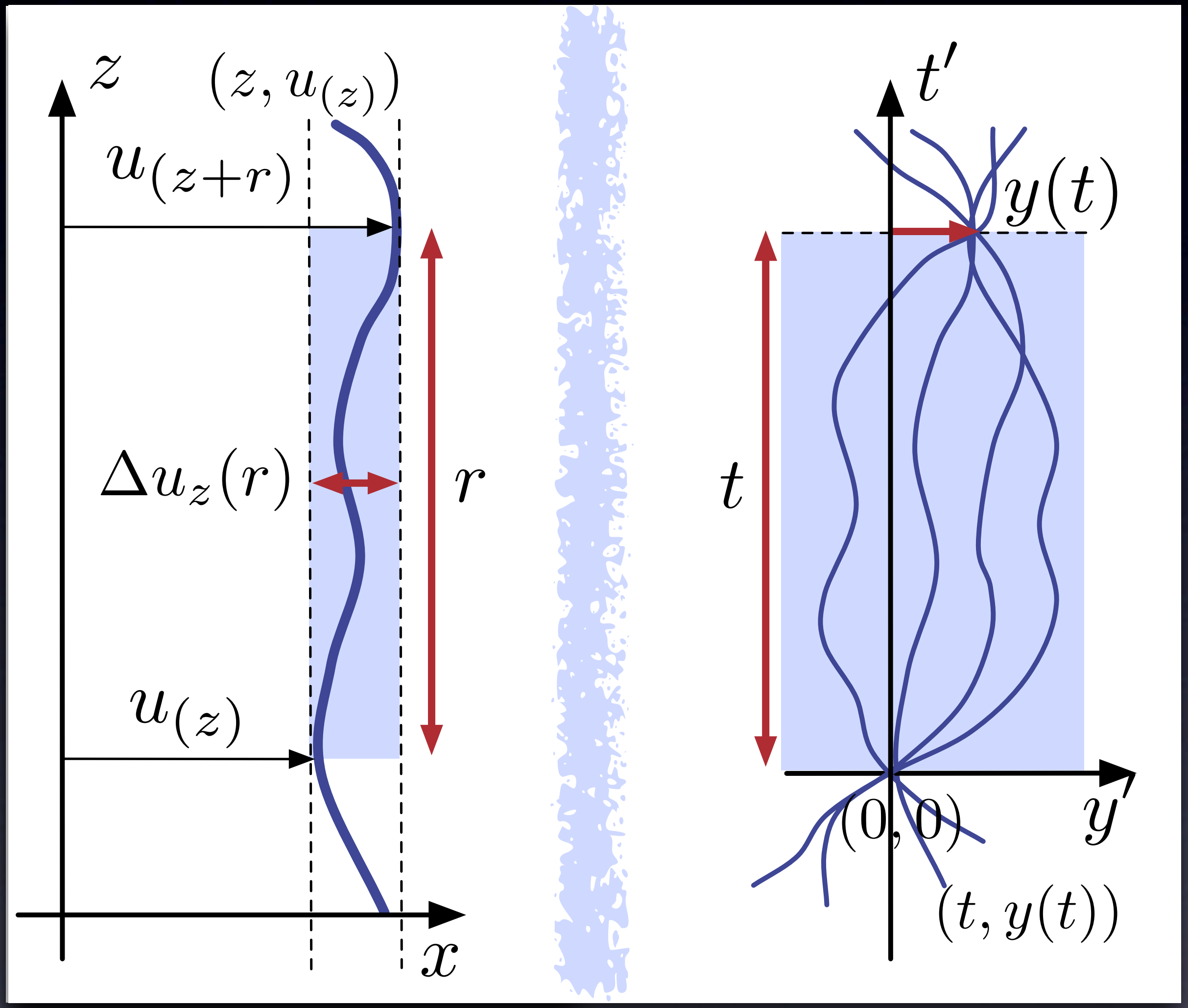
Static 1D interface & Growing 1+1 Directed Polymer (DP)

- *Observable*: static geometrical fluctuations $\mathcal{P}(\Delta u(r))$ & roughness $B(r) = \overline{\langle \Delta u(r)^2 \rangle}$

& Effective disorder experienced by the 1D interface at a given lengthscale r
$\leftrightarrow$ at fixed growing DP 'time' t



Integrating the thermal fluctuations
at short-'times'/lengthscales!



1D interface Lengthscale

Directed Polymer Growing 'time'

Static 1D interface & Growing 1+1 Directed Polymer (DP)

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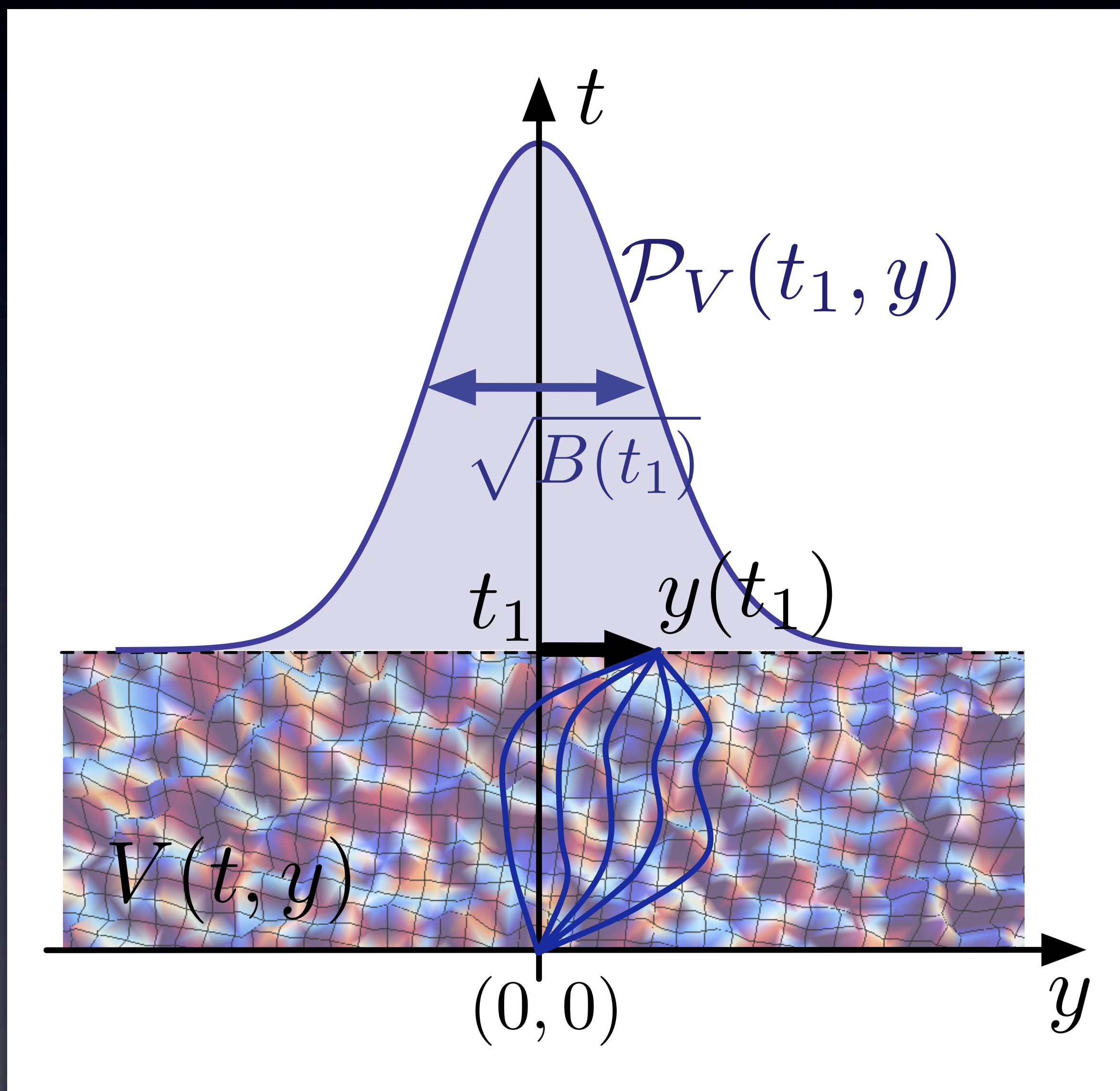
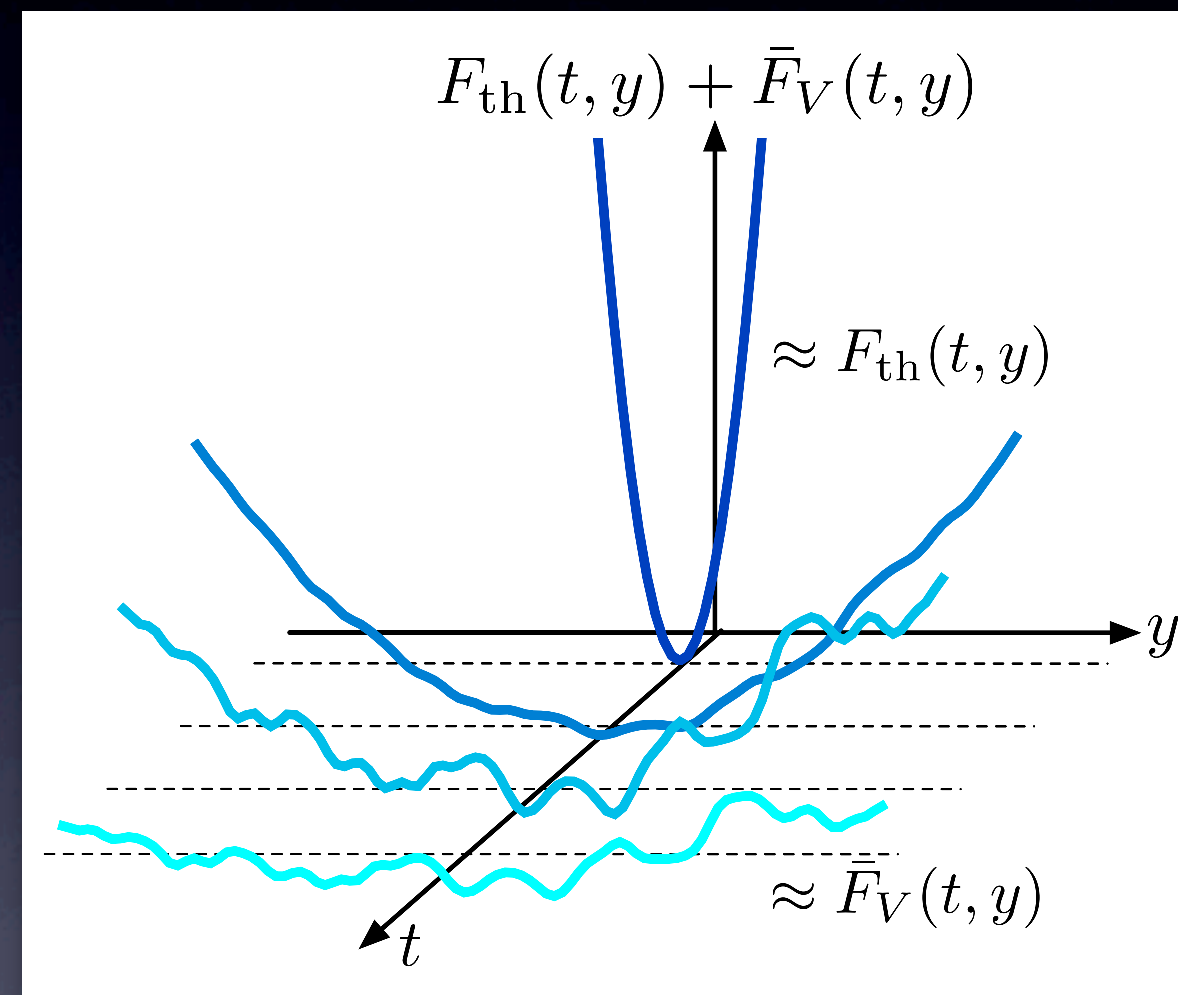


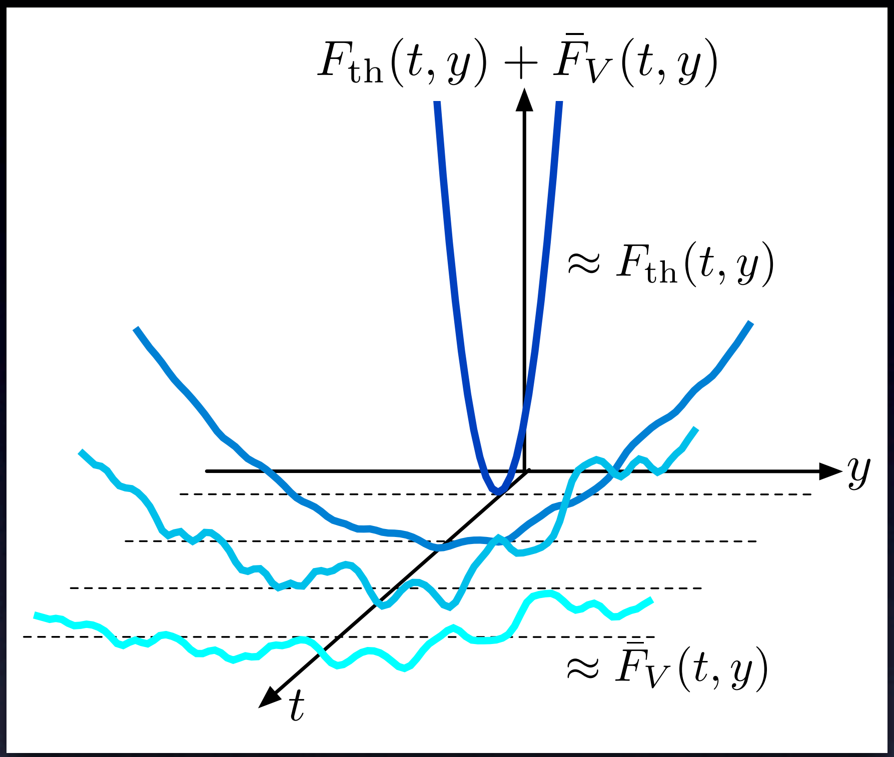


$$\mathcal{P}_V(t,y) \propto e^{-\left[F_{\text{th}}(t,y)+\bar{F}_V(t,y)\right]/T}$$

Integrating the thermal fluctuations
at short-'times'/lengthscales! $\Rightarrow$ 'Time-dependent free-energy landscape

Static 1D interface & Growing 1+1 Directed Polymer (DP)

- KPZ evolution equation for the total free-energy with 'sharp wedge' initial condition



D. Huse, C. L. Henley & D. S. Fisher, *Phys. Rev. Lett.* **55** 2924 (1985).
M. Kardar, G. Parisi & Y.-C. Zhang, *Phys. Rev. Lett.* **56** 889 (1986).

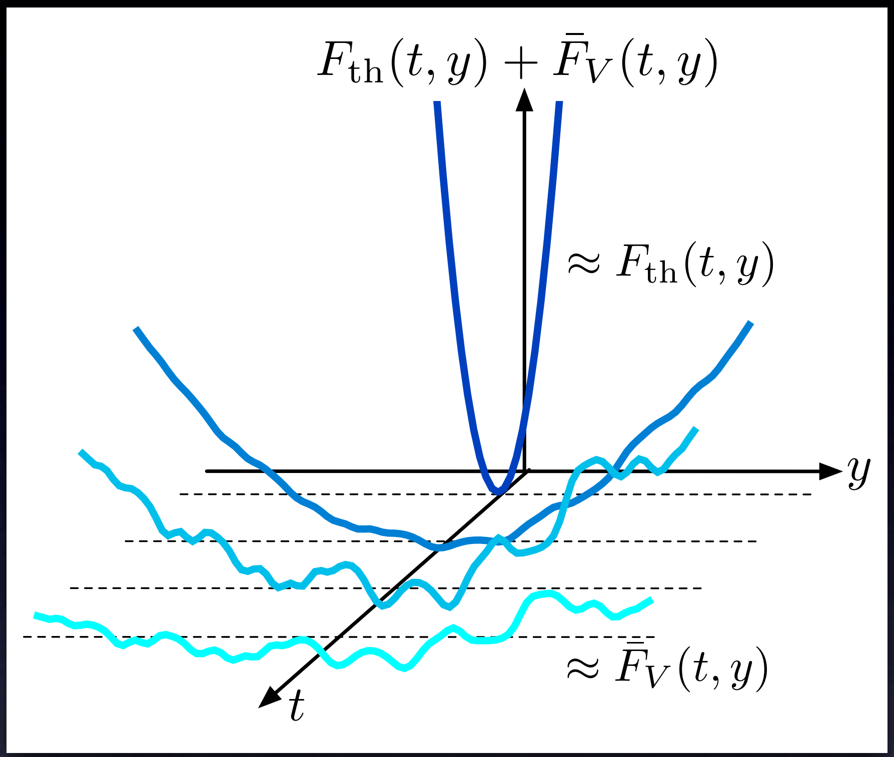
$$\begin{cases} \partial_t F_V(t,y) = \frac{T}{2c}\partial_y^2 F_V(t,y) - \frac{1}{2c}\left[\partial_y F_V(t,y)\right]^2 + V(t,y) \\ \mathcal{P}_V(0,y) = e^{-F_V(0,y)/T} = \delta(y) \end{cases}$$

- Tilted KPZ equation for the disorder contribution to the free-energy

E. Agoritsas, V. Lecomte & T. Giamarchi, Phys. Rev. E ***87****, 042406 & 062405 (2013).*

$$\begin{cases} \partial_t \bar{F}_V(t,y) = \frac{T}{2c}\partial_y^2 \bar{F}_V(t,y) - \frac{1}{2c}\left[\partial_y \bar{F}_V(t,y)\right]^2 - \frac{y}{t}\partial_y \bar{F}_V(t,y) + V(t,y) \\ \bar{F}_V(0,y) \equiv 0 \quad \text{('flat' initial condition)} \end{cases}$$

Static 1D interface & Growing 1+1 Directed Polymer (DP)



- Focus on the unknown part of the free-energy
- Translation-invariant distribution:
$$\bar{\mathcal{P}}\left[\bar{F}_V(t,y+Y)\right] = \bar{\mathcal{P}}\left[\bar{F}_V(t,y)\right]$$
- Starting point of numerical/analytical study

- Tilted KPZ equation for the disorder contribution to the free-energy:

*E. Agoritsas, V. Lecomte & T. Giamarchi, Phys. Rev. E **87**, 042406 & 062405 (2013).*

$$\begin{cases} \partial_t \bar{F}_V(t,y) = \dfrac{T}{2c}\partial_y^2 \bar{F}_V(t,y) - \dfrac{1}{2c}\left[\partial_y \bar{F}_V(t,y)\right]^2 - \dfrac{y}{t}\partial_y \bar{F}_V(t,y) + V(t,y) \\ \bar{F}_V(0,y) \equiv 0 \quad \text{('flat' initial condition)} \end{cases}$$

Kardar-Parisi-Zhang (KPZ) equation

M. Kardar, G. Parisi & Y.-C. Zhang, « Dynamical Scaling of Growing Interfaces », *Phys. Rev. Lett.* **56** 889 (1986).

- Model for the time-evolution of the profile of a growing interface $h(t,\vec{x}) \leftrightarrow F_V(t,y)$

$$\partial_t h(t,\vec{x}) = \underbrace{\nu\, \nabla^2_{\vec{x}} h(t,\vec{x})}_{\text{relaxation or diffusion}} + \underbrace{\frac{\lambda}{2}\, \left[\nabla_{\vec{x}} h(t,\vec{x})\right]^2}_{\text{slope-dependent lateral growth}} + \underbrace{\eta(t,\vec{x})}_{\text{random noise}}$$

Initial condition $h(0,\vec{x})$

Distribution of the random noise $\bar{\mathcal{P}}\left[\eta(t,\vec{x})\right]$

- Gaussian statistical distribution of a ***white*** noise...

$$\begin{cases} \overline{\eta(t,\vec{x})} = 0 \\ \overline{\eta(t,\vec{x})\eta(t',\vec{x}')} = D \cdot \delta(t-t') \cdot \delta^{(d)}(\vec{x}-\vec{x}') \end{cases}$$

... and of a **colored** noise in 1D (d=1)

$$\overline{\eta(t,x)\eta(t',x')} = D \cdot \delta(t-t') \cdot R_\xi(x-x')$$

Kardar-Parisi-Zhang (KPZ) equation

$$\partial_t h(t,\vec{x}) = \nu\, \nabla_{\vec{x}}^2 h(t,\vec{x}) + \frac{\lambda}{2}\, \left[\nabla_{\vec{x}} h(t,\vec{x})\right]^2 + \eta(t,\vec{x})$$

- 1D KPZ universality class encompasses a wide range of problems:

Random matrices, Burgers equation in hydrodynamics, roughening phenomena & stochastic growth, 1+1 Directed Polymer (DP), our one-dimensional interface, ...

Fluctuations with power-law of exponent $\zeta_{\mathrm{KPZ}} = 2/3$

- Ivan Corwin, « The Kardar-Parisi-Zhang equation and universality class »

Random Matrices: Theory and Applications, **1** 1130001 (2012), http://arxiv.org/abs/1106.1596.

Jeremy Quastel, « Introduction to KPZ »,

Lecture notes of the 2012 Arizona School of Analysis and Mathematical Physics (http://math.arizona.edu/~mathphys/school_2012/IntroKPZ-Arizona.pdf)

> 2.7. **KPZ universality, or universality of KPZ?.** All of the models described in this section, as well as the KPZ equation, are believed to be members of the *KPZ universality class*, in the sense that they should have the scaling exponent $z = 3/2$, and, at a more refined level, the correct fluctuations (GUE/Airy$_2$, GOE/Airy$_1$, Baik-Rains/Brownian motion) at the scale (31) depending on the initial conditions (curved, flat, equilibrium). We have sketched above what is proved for special cases.

Free-energy of the 1+1 DP: uncorrelated disorder $(\xi = 0)$

- Free-energy two-point correlators: $\begin{cases} \bar{C}(t,y) \equiv \overline{\left[\bar{F}_V(t,y) - \bar{F}_V(t,0)\right]^2} \\ \bar{R}(t,y) \equiv \overline{\partial_y \bar{F}_V(t,y)\, \partial_y \bar{F}_V(t,0)} \end{cases}$

- *Uncorrelated* disorder (white-noise): $\boxed{R_{\xi=0}(y) = \delta(y)}$

- *Infinite-'time' limit:* $\begin{cases} \text{Gaussian distribution} \\ \bar{C}(\infty,y) = \frac{cD}{T}\,|y| \iff \bar{R}(\infty,y) = \frac{cD}{T} R_{\xi=0}(y) \end{cases}$

D. A. Huse, C. L. Henley & D. S. Fisher, *Phys. Rev. Lett.* **55** 2294 (1985).

- *Asymptotically large-'time':* $\begin{cases} \text{GUE Tracy-Widom distribution (non-Gaussian!)} \\ \bar{C}(t,y) = \text{2-point correlator of Airy}_2 \text{ process} \end{cases}$

M. Prähofer & H. Spohn, *J. Stat. Phys.* **159** 1071 (2002).

- *At all 'times':*

P. Calabrese, P. Le Doussal & A. Rosso, *Eur. Phys. Lett.* **90** 20002 (2010).
V. Dotsenko, *Eur. Phys. Lett.* **90** 20003 (2010).
T. Sasamoto & H. Spohn, *Nucl. Phys. B* **834** 523 (2010).
G. Amir, I. Corwin, J. Quastel., *Comm. Pure Appl. Math.* **64** 466 (2011).

Free-energy of the 1+1 DP: 'time'-dependence $(\xi > 0)$

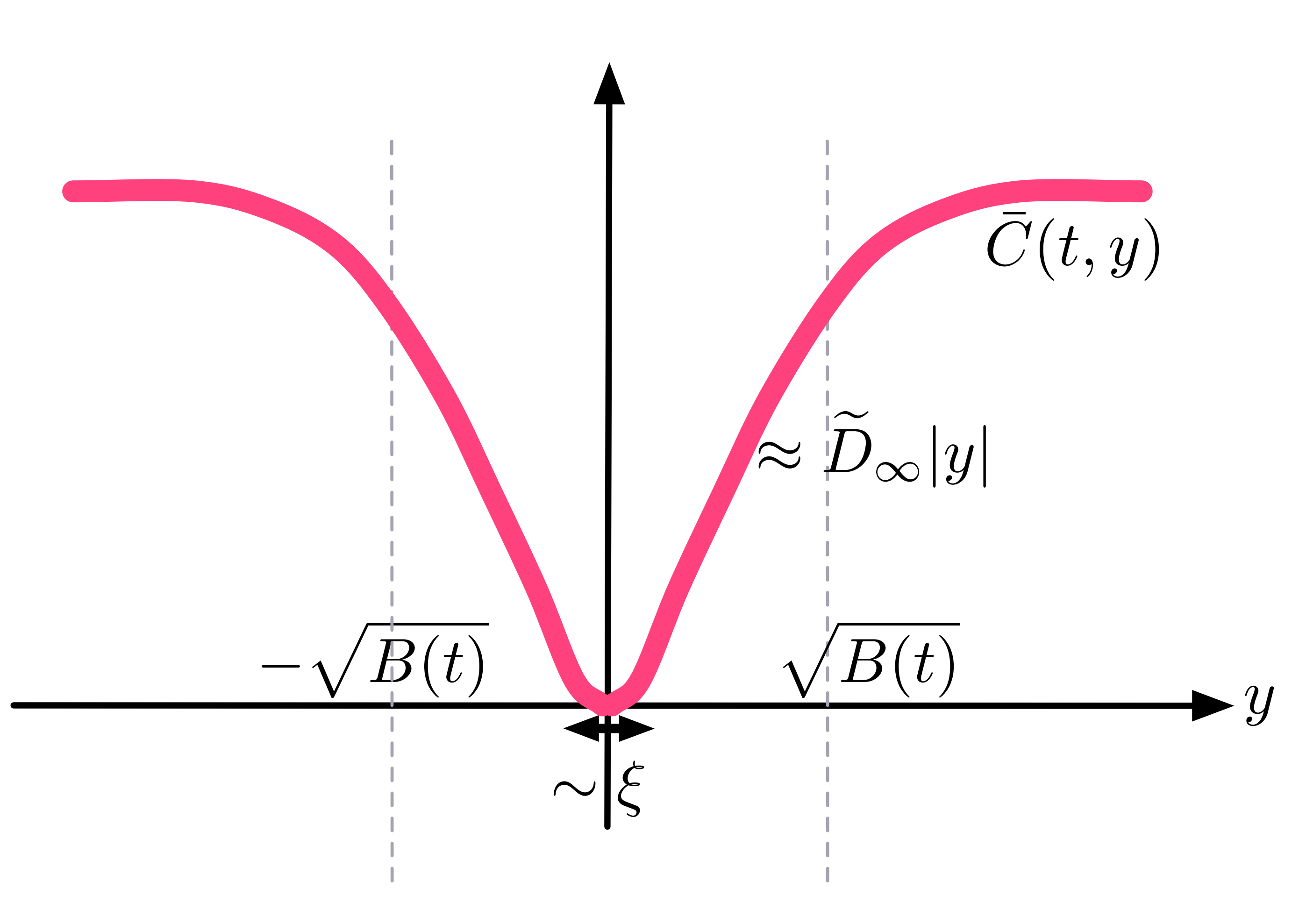
- Free-energy two-point correlators:

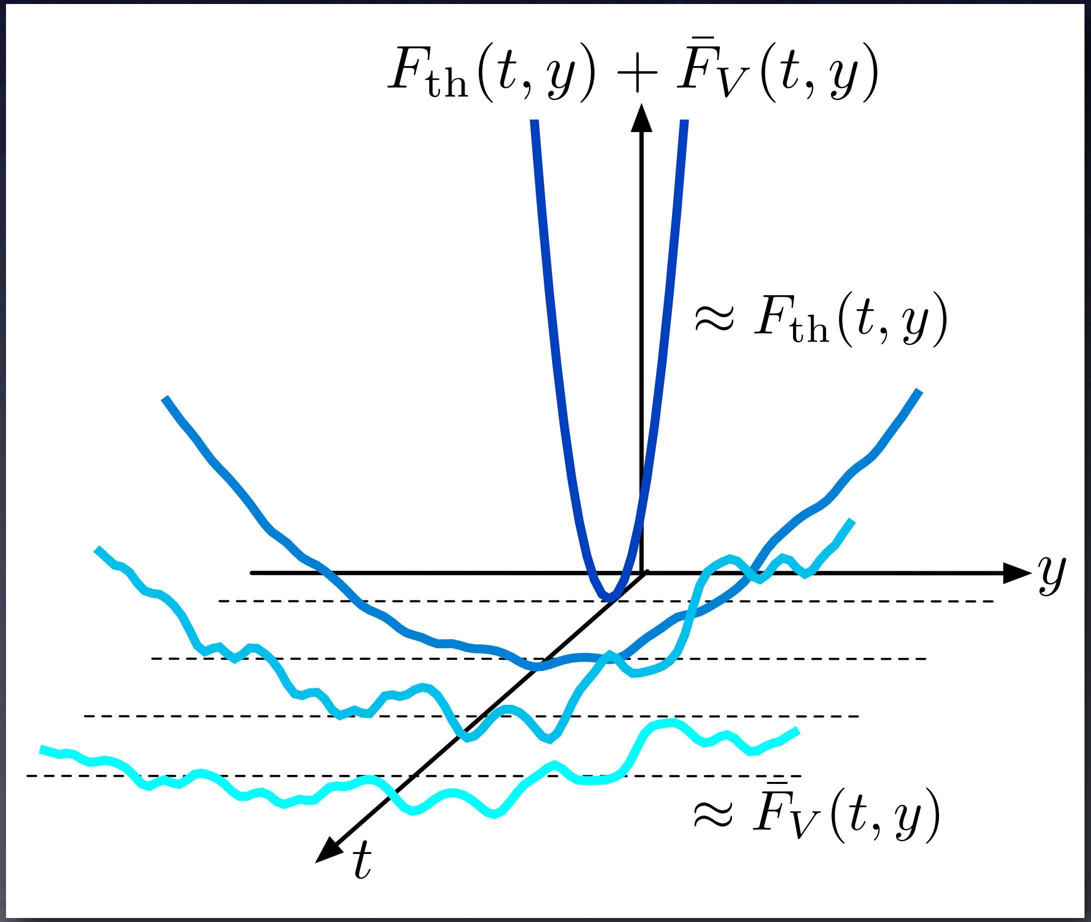
$$\begin{cases} \bar{C}(t,y) \equiv \overline{\left[\bar{F}_V(t,y) - \bar{F}_V(t,0)\right]^2} \\ \bar{R}(t,y) \equiv \overline{\partial_y \bar{F}_V(t,y)\, \partial_y \bar{F}_V(t,0)} \end{cases}$$

- *Correlated* disorder (colored-noise):

$$\boxed{R_\xi(y) = \xi^{-1} R_1(y/\xi)}$$

$$\overline{V(t,y)V(t',y')} = D \cdot \delta_{(t-t')}\, R_\xi(y-y')$$





E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. E* **87**, 042406 (2013).

Free-energy of the 1+1 DP: 'time'-dependence $(\xi > 0)$

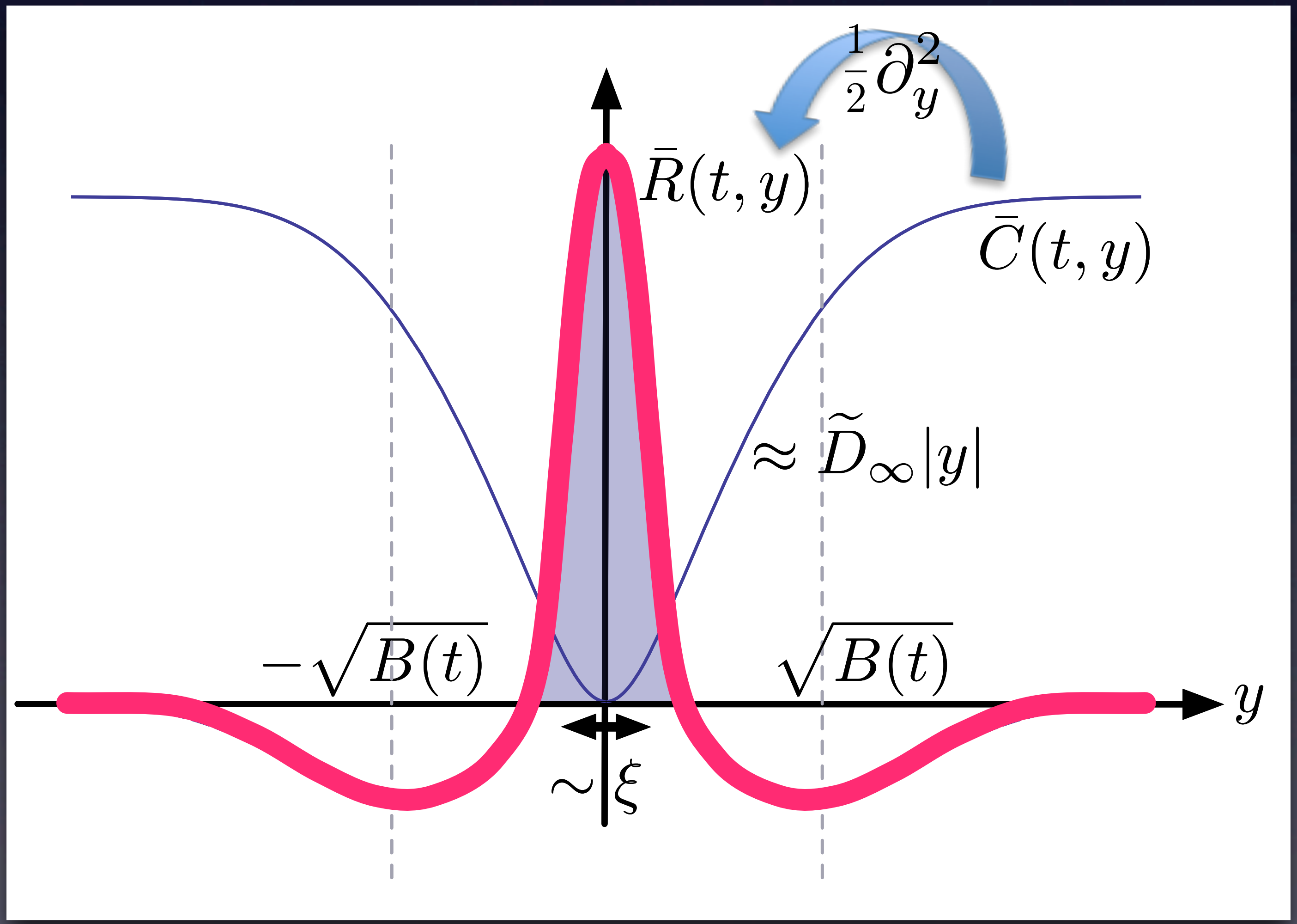
- Free-energy two-point correlators:

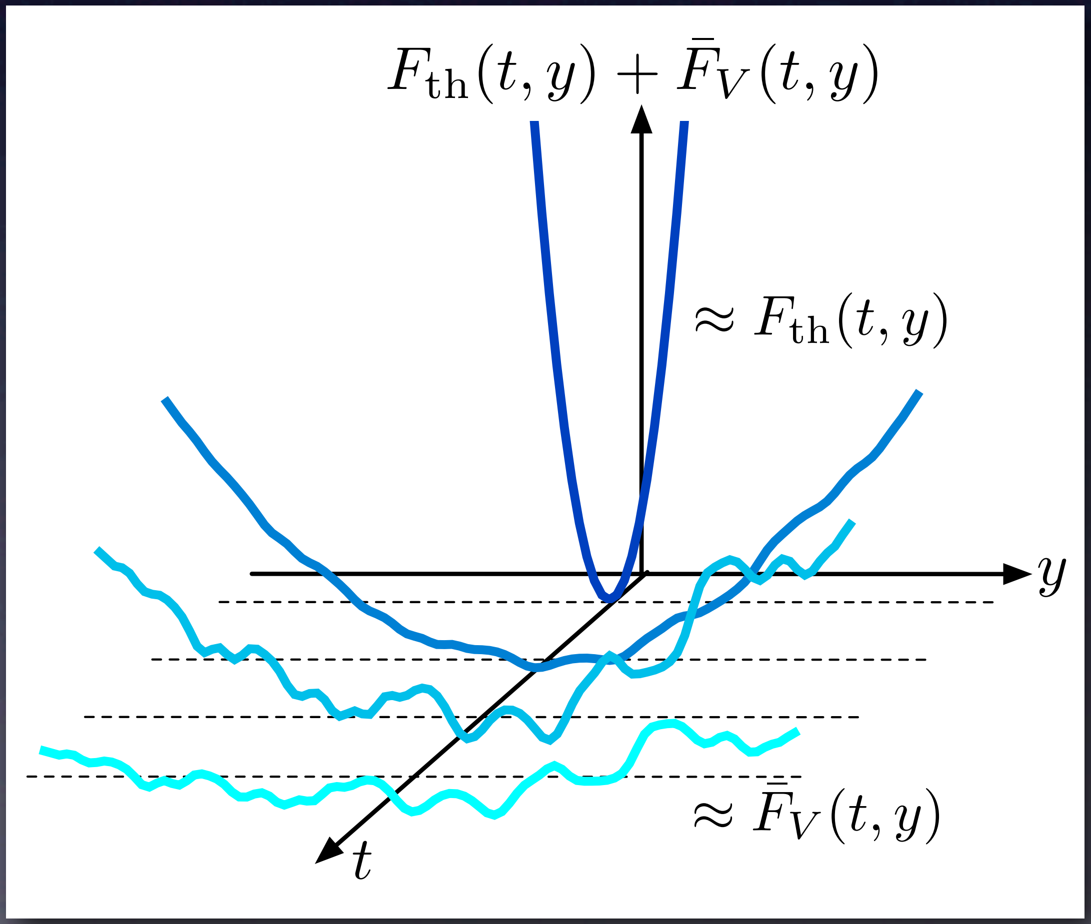
$$\begin{cases} \bar{C}(t,y) \equiv \overline{\left[\bar{F}_V(t,y) - \bar{F}_V(t,0)\right]^2} \\ \bar{R}(t,y) \equiv \overline{\partial_y \bar{F}_V(t,y)\, \partial_y \bar{F}_V(t,0)} \end{cases}$$

- *Correlated* disorder (colored-noise):

$$\boxed{R_\xi(y) = \xi^{-1} R_1(y/\xi)}$$

$$\overline{V(t,y)V(t',y')} = D \cdot \delta_{(t-t')}\, R_\xi(y-y')$$





E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. E* **87**, 042406 (2013).

Free-energy of the 1+1 DP: 'time'-dependence $(\xi > 0)$

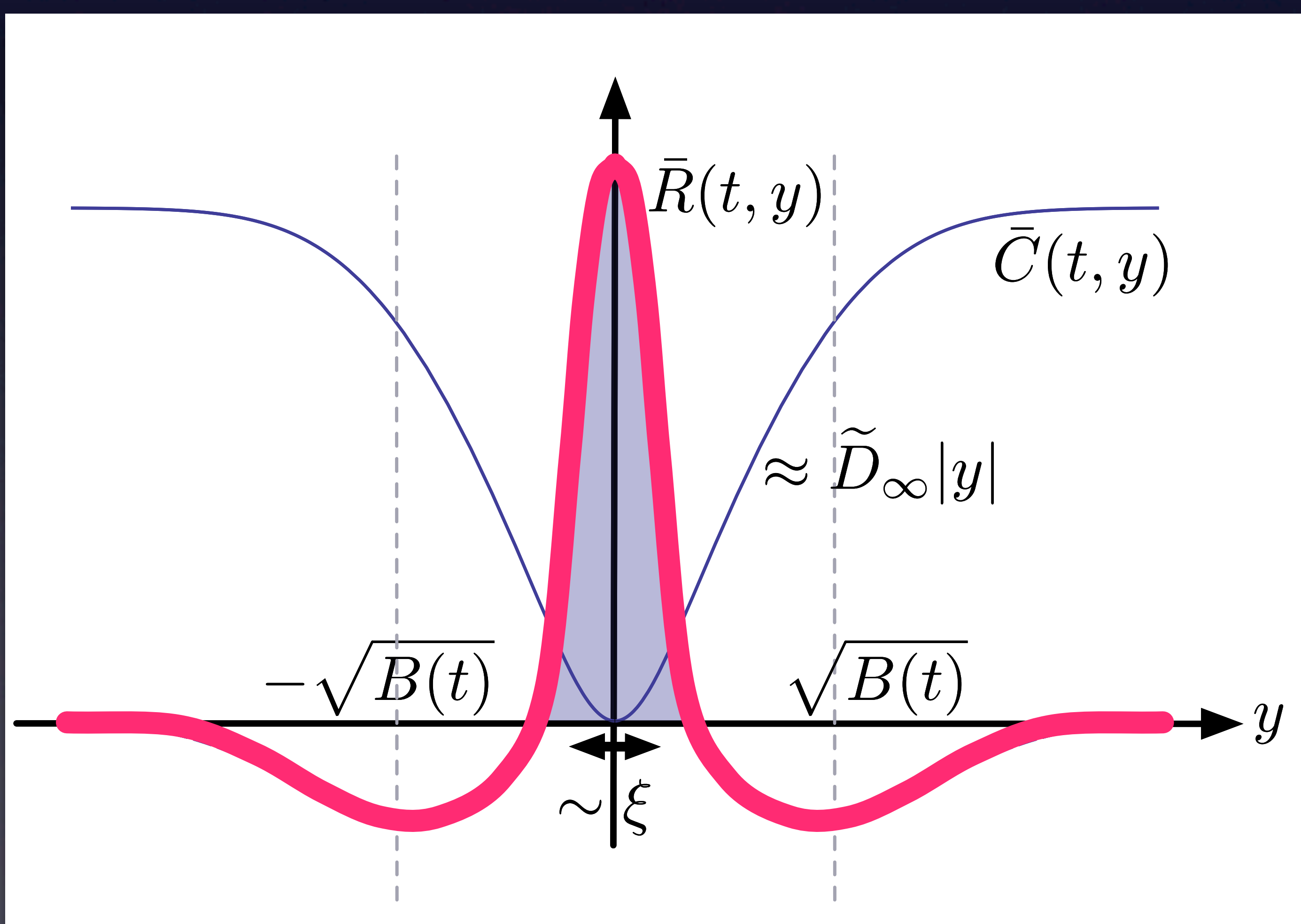
- Focus on the two-point correlators:

$$\begin{cases} \bar{C}(t,y) \equiv \overline{\left[\bar{F}_V(t,y) - \bar{F}_V(t,0)\right]^2} \\ \bar{R}(t,y) \equiv \overline{\partial_y \bar{F}_V(t,y)\, \partial_y \bar{F}_V(t,0)} \end{cases}$$

- *Correlated* disorder (colored-noise):

$$R_\xi(y) = \xi^{-1} R_1(y/\xi)$$

$$\bar{R}(t,y) = \widetilde{D}_\infty \left[\mathcal{R}_\xi(y) - b(t,y,\xi)\right]$$



Amplitude $\widetilde{D}_\infty(T,\xi)$

Shape: Microscopic disorder correlator?

Time dependence: Encoding the roughness?

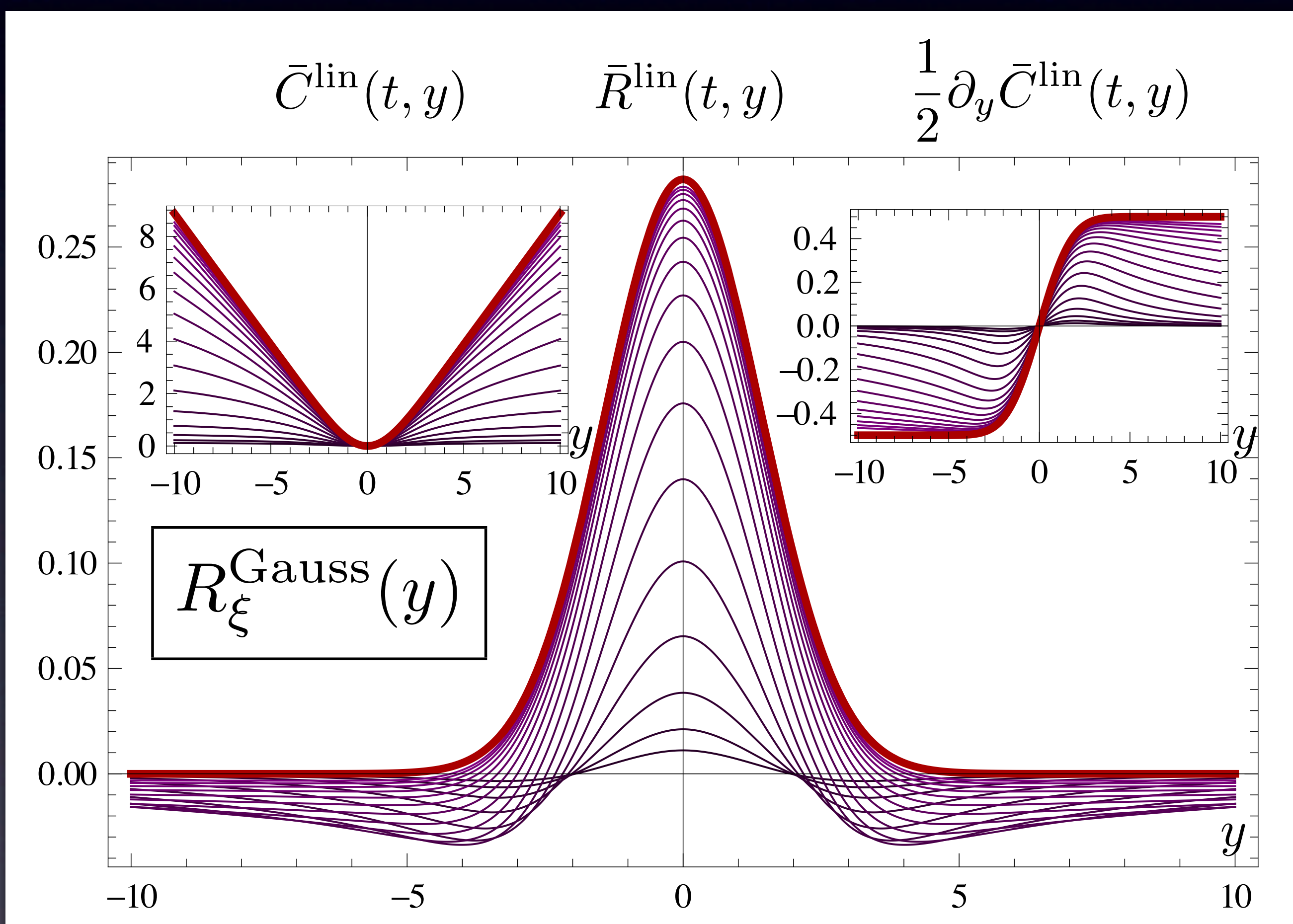
E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. E* **87**, 042406 (2013).

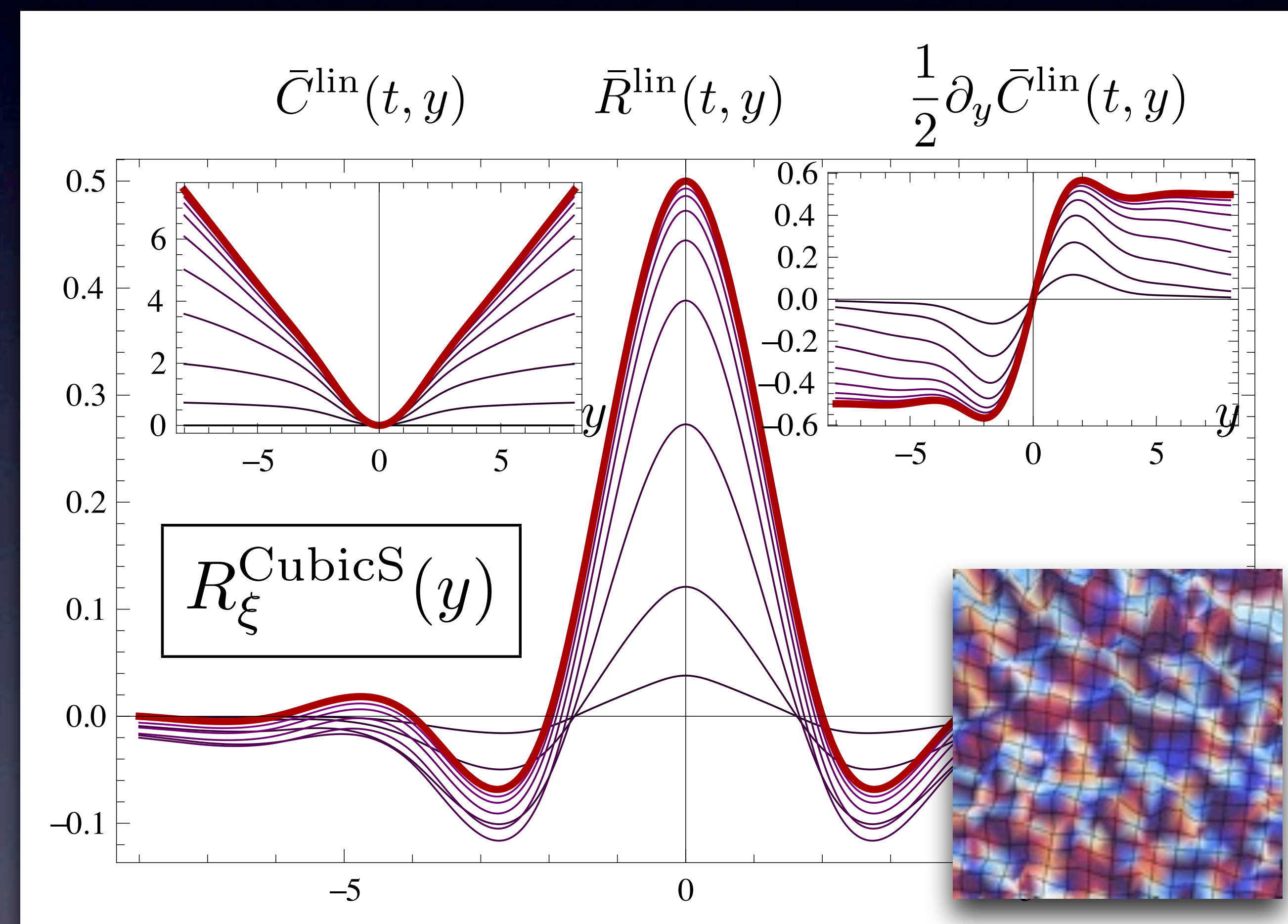
Free-energy of the 1+1 DP: linearized evolution

$$\partial_t \bar{F}_V(t,y) = \frac{T}{2c}\partial_y^2 \bar{F}_V(t,y) - \frac{1}{2c}\left[\partial_y \bar{F}_V(t,y)\right]^2 - \frac{y}{t}\partial_y \bar{F}_V(t,y) + V(t,y)$$

- Fluctuations are exactly Gaussian at all `times' ⇒ fully characterized by: $\bar{R} = \overline{\partial \bar{F} \partial \bar{F}}$

$$\bar{R}^{\text{lin}}(t,y) = \frac{cD}{T}\left[R_\xi(y) - b^{\text{lin}}(t,y,\xi)\right]$$



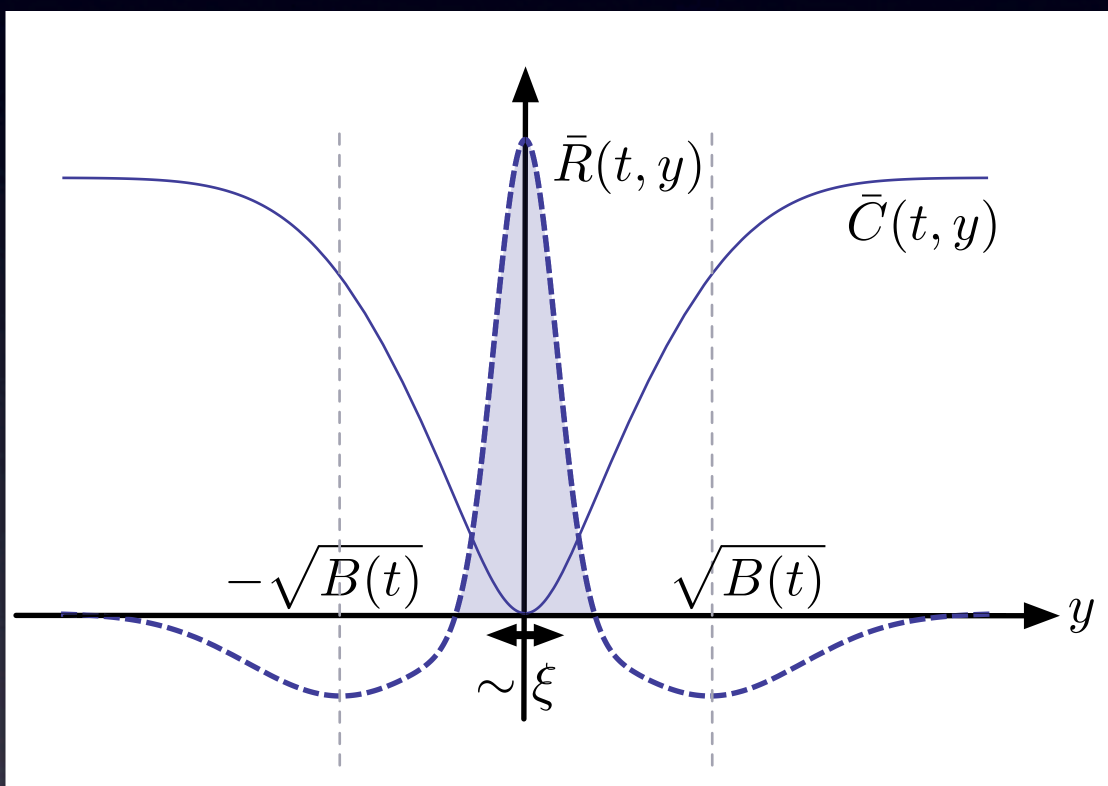


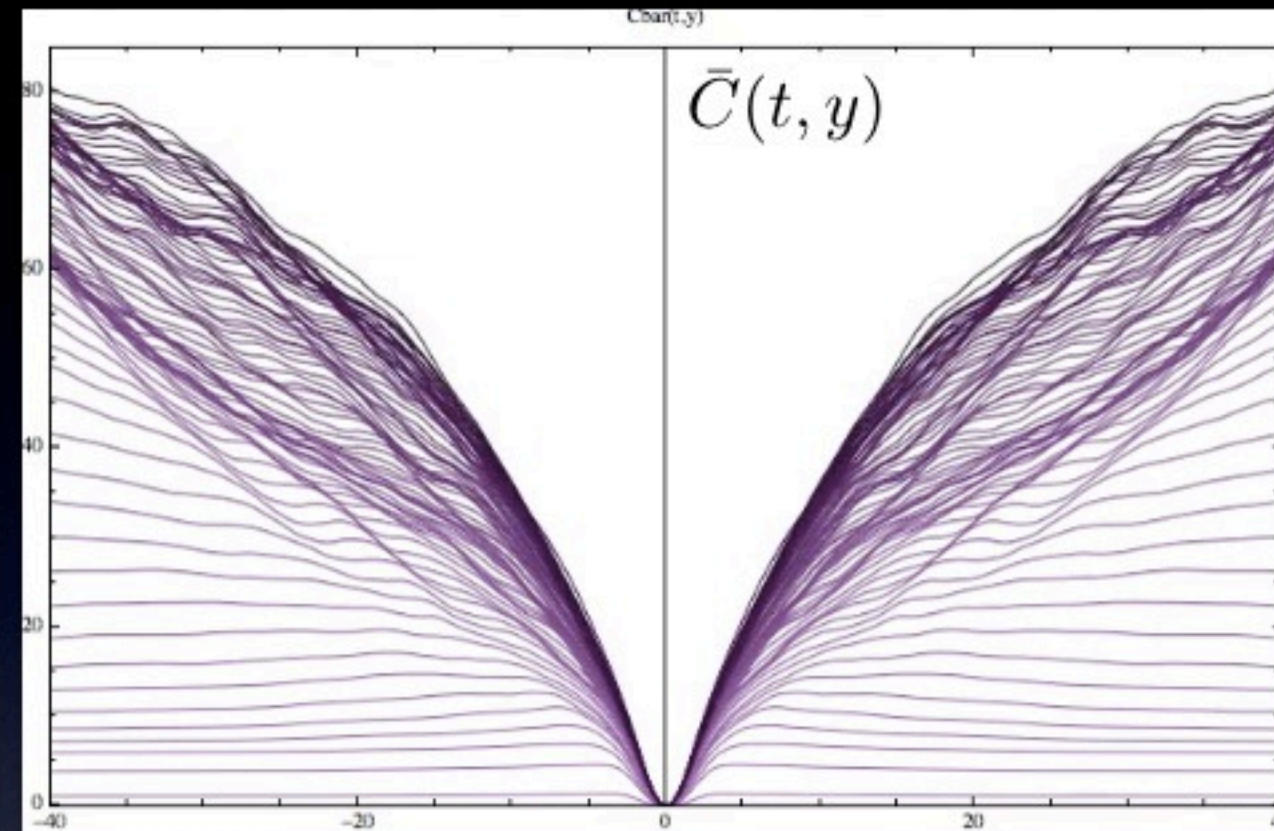
- Scaling with the diffusive roughness:

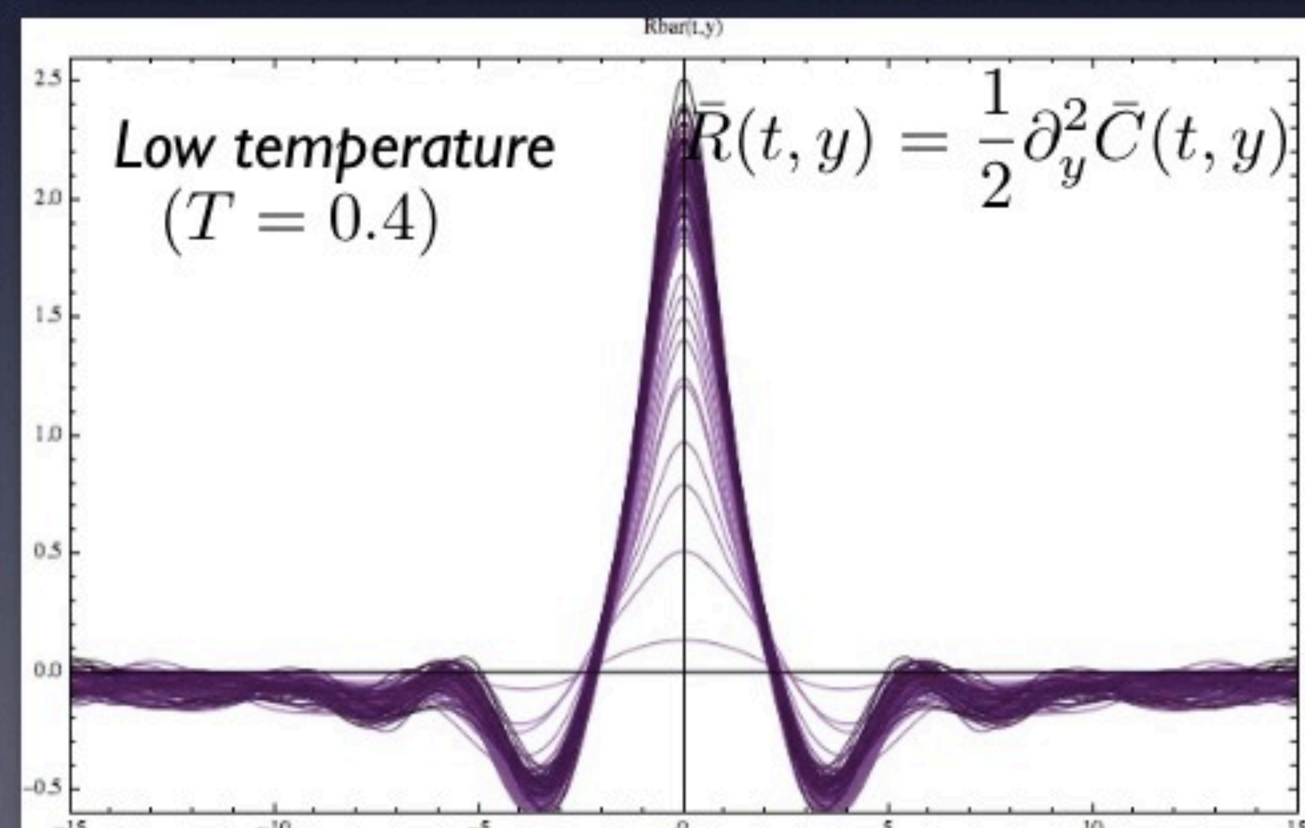
$$b^{\text{lin}}(t,y,\xi) = \frac{\tilde{b}(y/\sqrt{B_{\text{th}}(t)}, \xi/\sqrt{B_{\text{th}}(t)})}{\sqrt{B_{\text{th}}(t)}}$$

Numerics: 'time'-evolution of the free-energy correlators $(\xi > 0)$

$$\begin{cases} \bar{C}(t,y) \equiv \overline{\left[\bar{F}_V(t,y) - \bar{F}_V(t,0)\right]^2} \\ \bar{R}(t,y) \equiv \overline{\partial_y \bar{F}_V(t,y)\, \partial_y \bar{F}_V(t,0)} \end{cases}$$

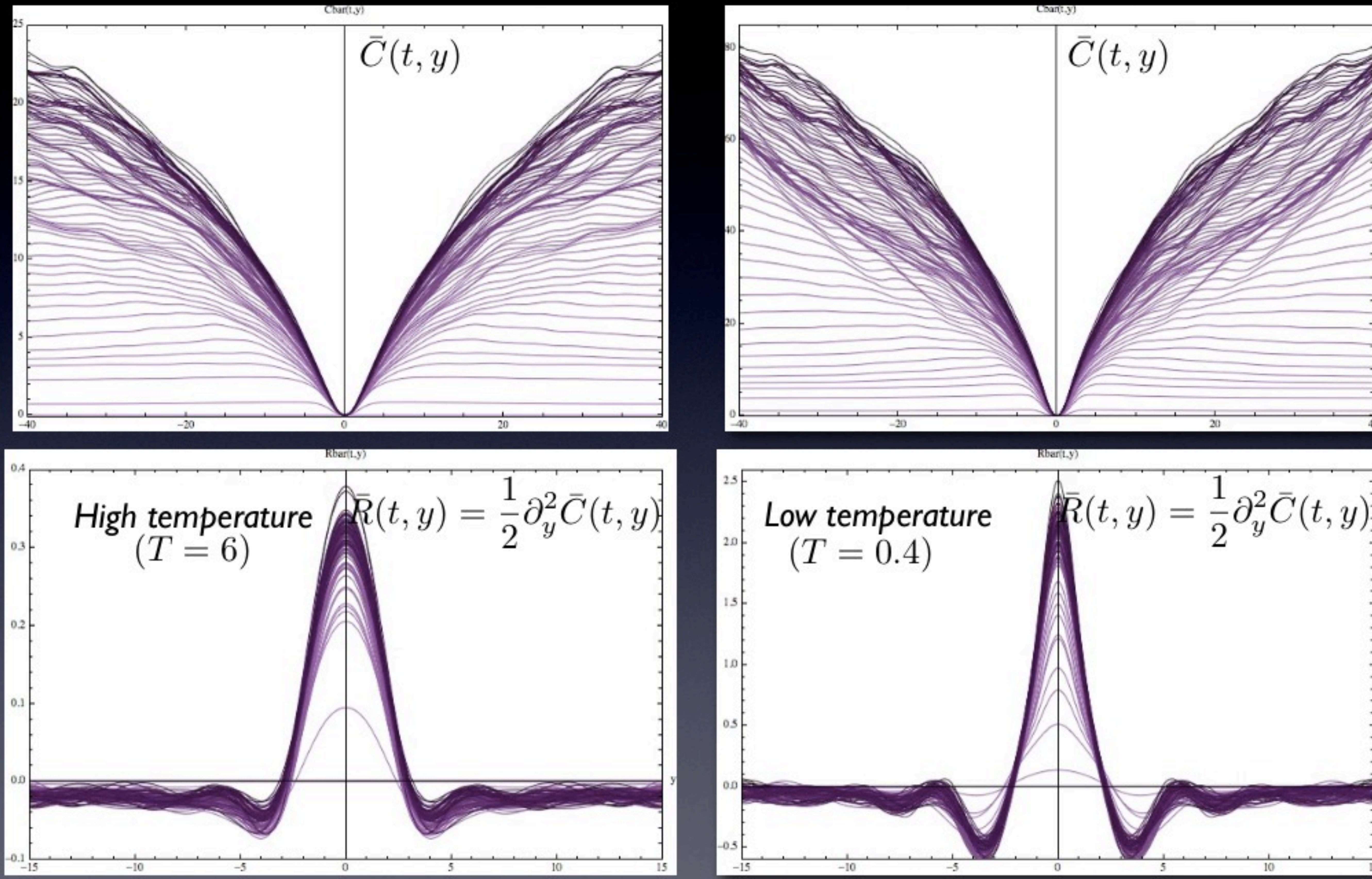






E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. E* **87**, 062405 (2013).

Numerics: 'time'-evolution of the free-energy correlators $(\xi > 0)$



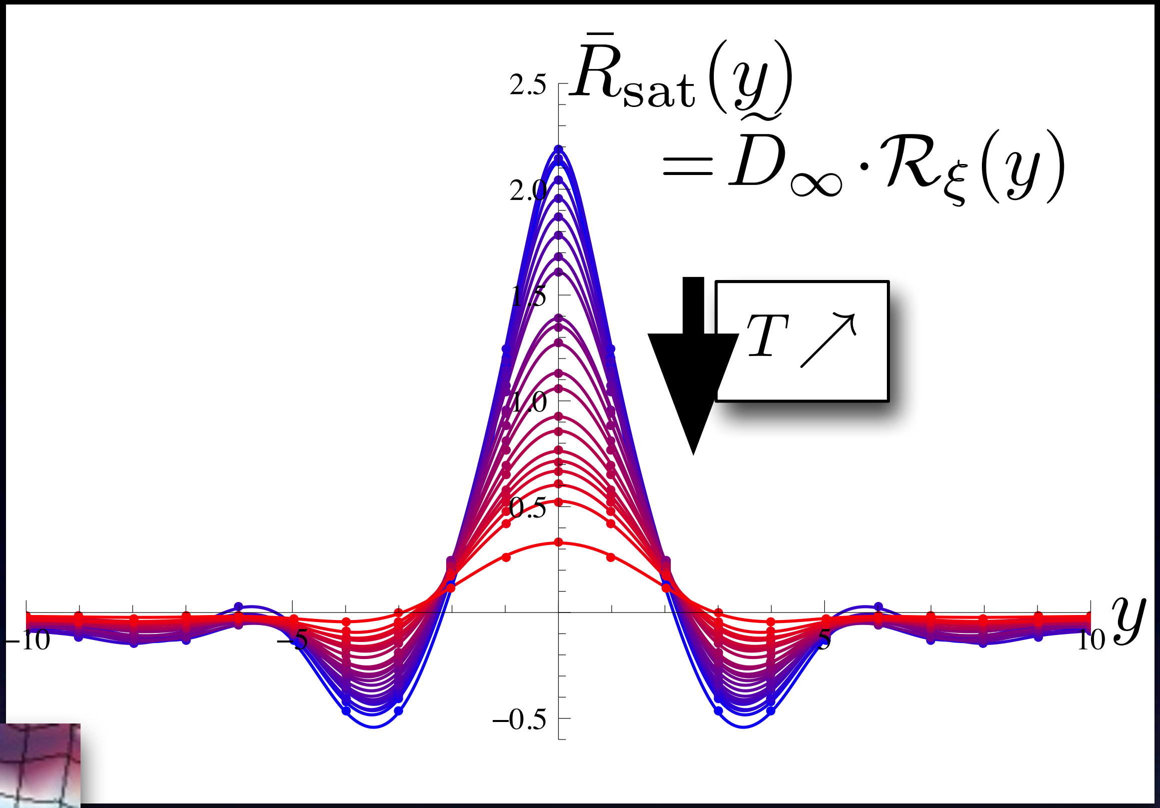
E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. E* **87**, 062405 (2013).

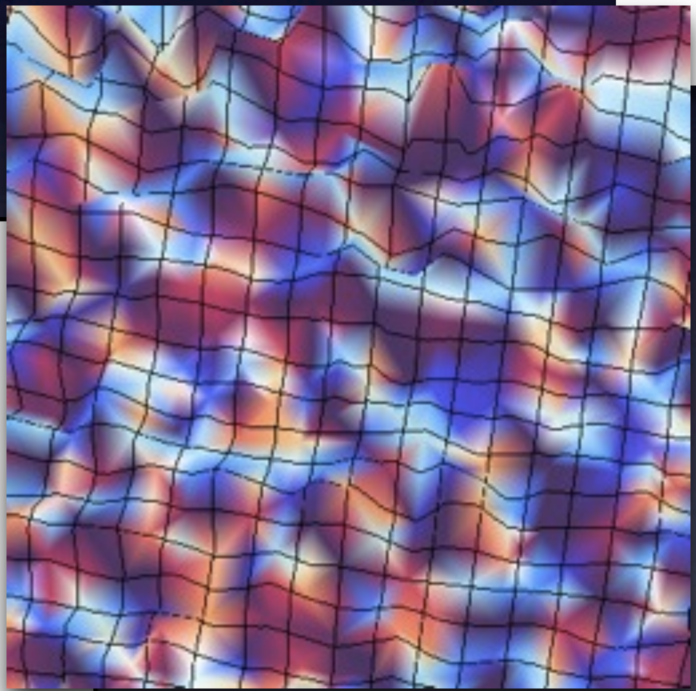
Numerics: shape of the asymptotic correlator $(\xi > 0)$

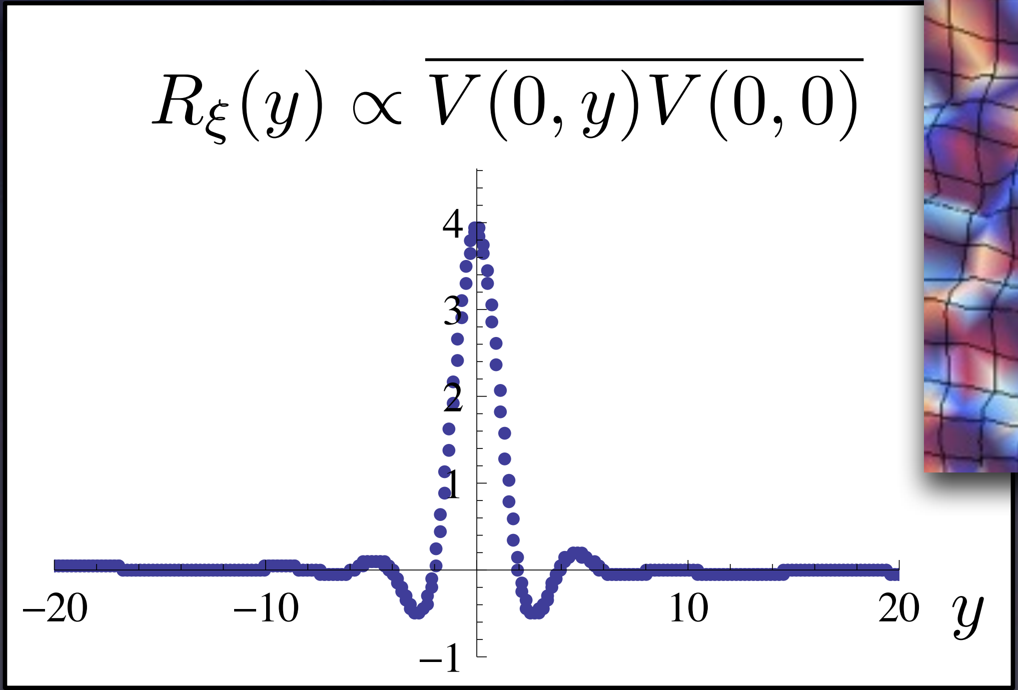
- Asymptotic disorder free-energy correlator

$$\bar{R}_{\text{sat}}(y) \approx \bar{R}(\infty, y) = \frac{1}{2}\partial_y^2 \bar{C}(\infty, y)$$

- Shape reminiscent of the microscopic disorder correlator used in our numerical study!







E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. B* **82**, 184207 (2010).
E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. E* **87**, 042406 & 062405 (2013).

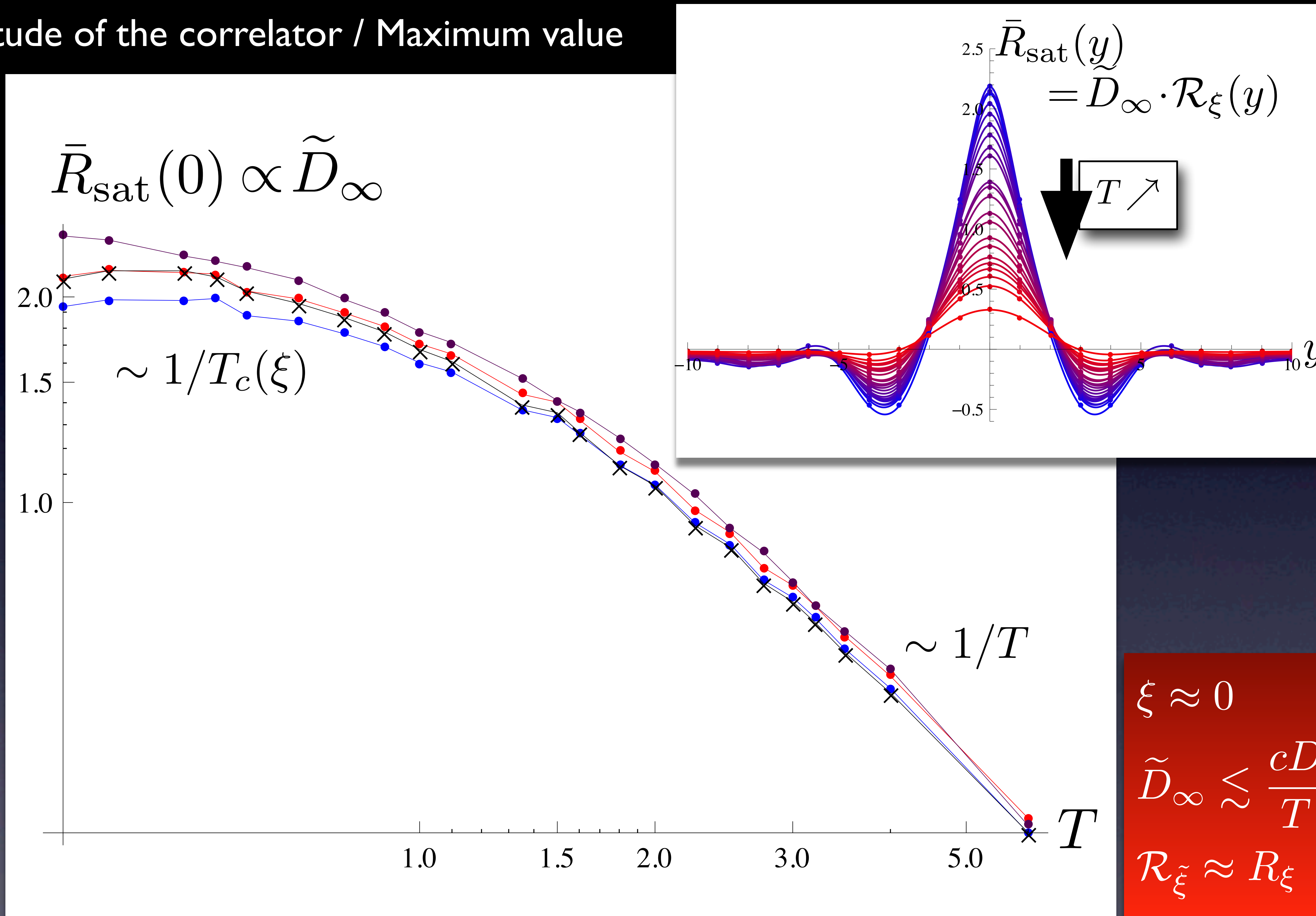
Numerics: temperature dependence of the free-energy $(\xi > 0)$

- Amplitude of the correlator / Maximum value

$T \approx 0$

$\widetilde{D}_\infty \sim \frac{cD}{T_c}$

$\mathcal{R}_{\tilde{\xi}} \approx ??$



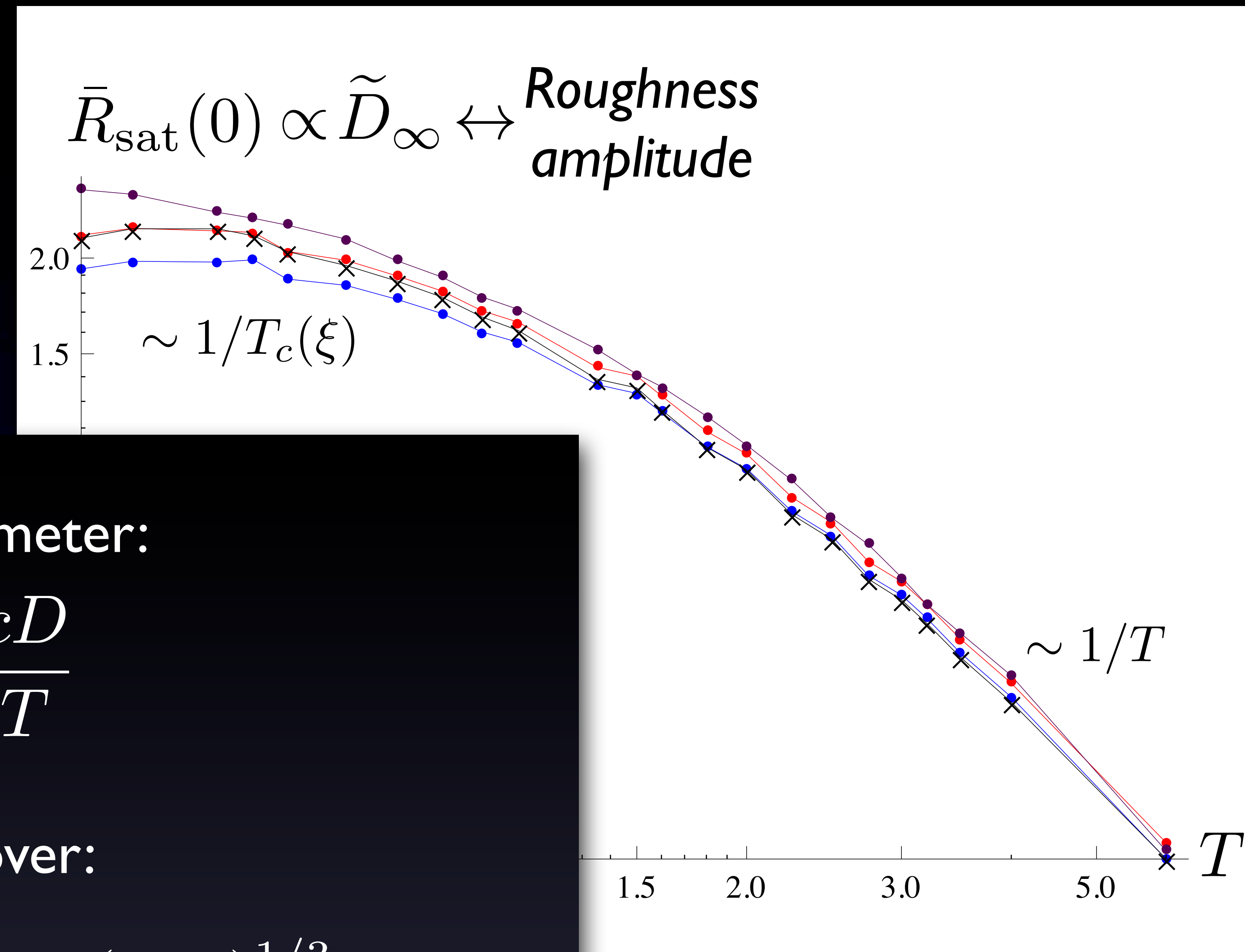
$\xi \approx 0$

$\widetilde{D}_\infty \lesssim \frac{cD}{T}$

$\mathcal{R}_{\tilde{\xi}} \approx R_\xi$

E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. B* **82**, 184207 (2010).
E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. E* **87**, 042406 & 062405 (2013).

Revisiting the GVM: analytical prediction for the crossover $(\xi > 0)$



- Definition of the interpolating parameter:

$$\widetilde{D}_\infty(T,\xi) = f(T,\xi)\frac{cD}{T}$$

- GVM prediction of a smooth crossover:

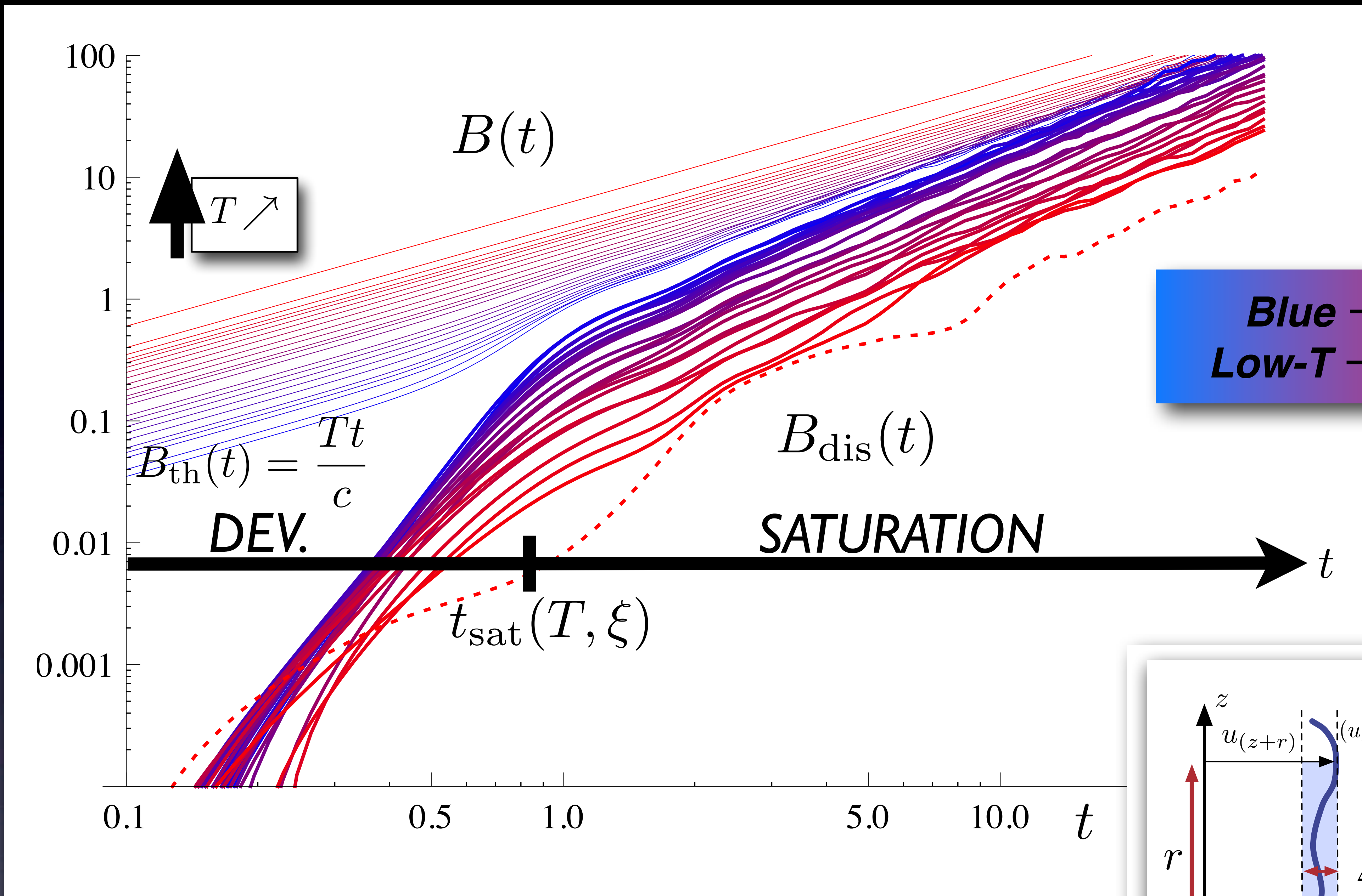
$$f^6 \propto (T/T_c)^6(1-f) \quad \& \quad T_c(\xi) = (\xi c D)^{1/3}$$

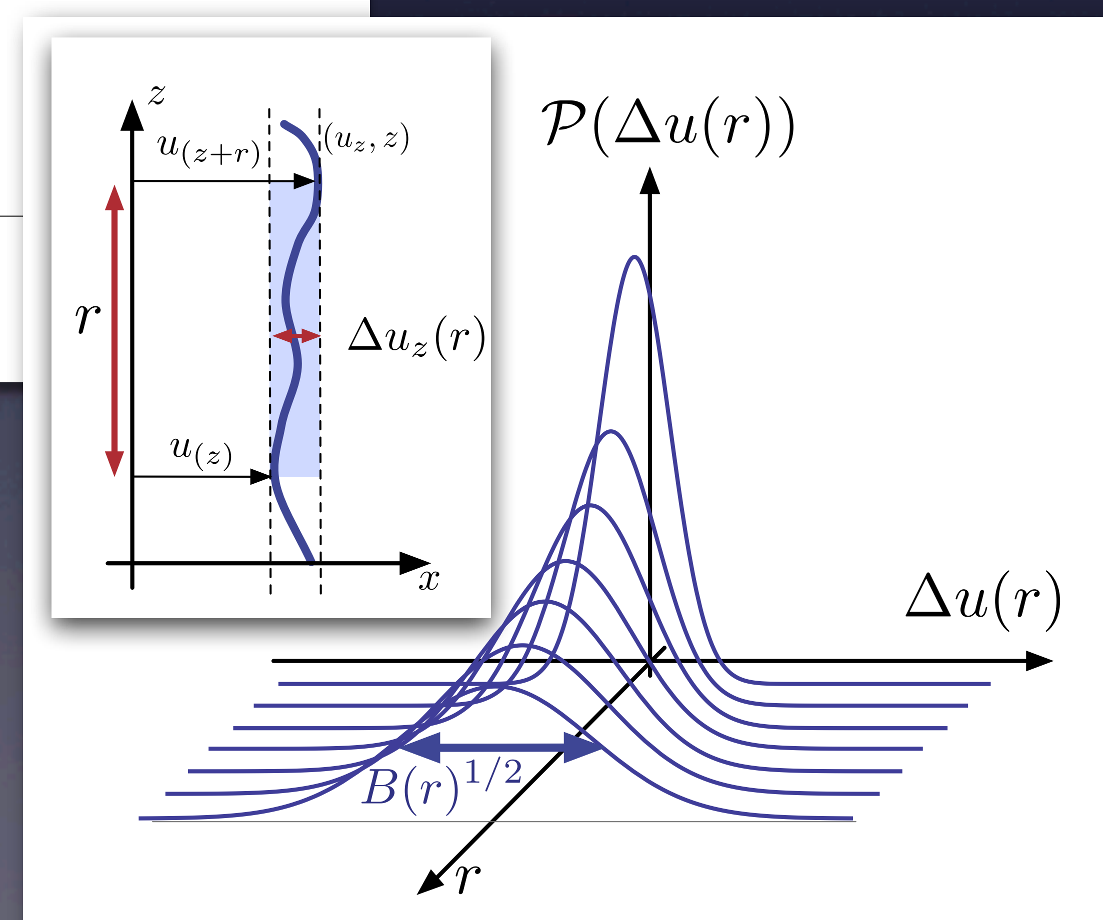
- Connexion with the asymptotic roughness amplitude:

$$A_{(c,D,T,\xi)} \sim (\widetilde{D}_\infty/c^2)^{2/3}$$

E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. B* **82**, 184207 (2010).
E. Agoritsas, V. Lecomte & T. Giamarchi, *Phys. Rev. E* **87**, 042406 & 062405 (2013).

Numerics: disorder contribution to the roughness $(\xi > 0)$

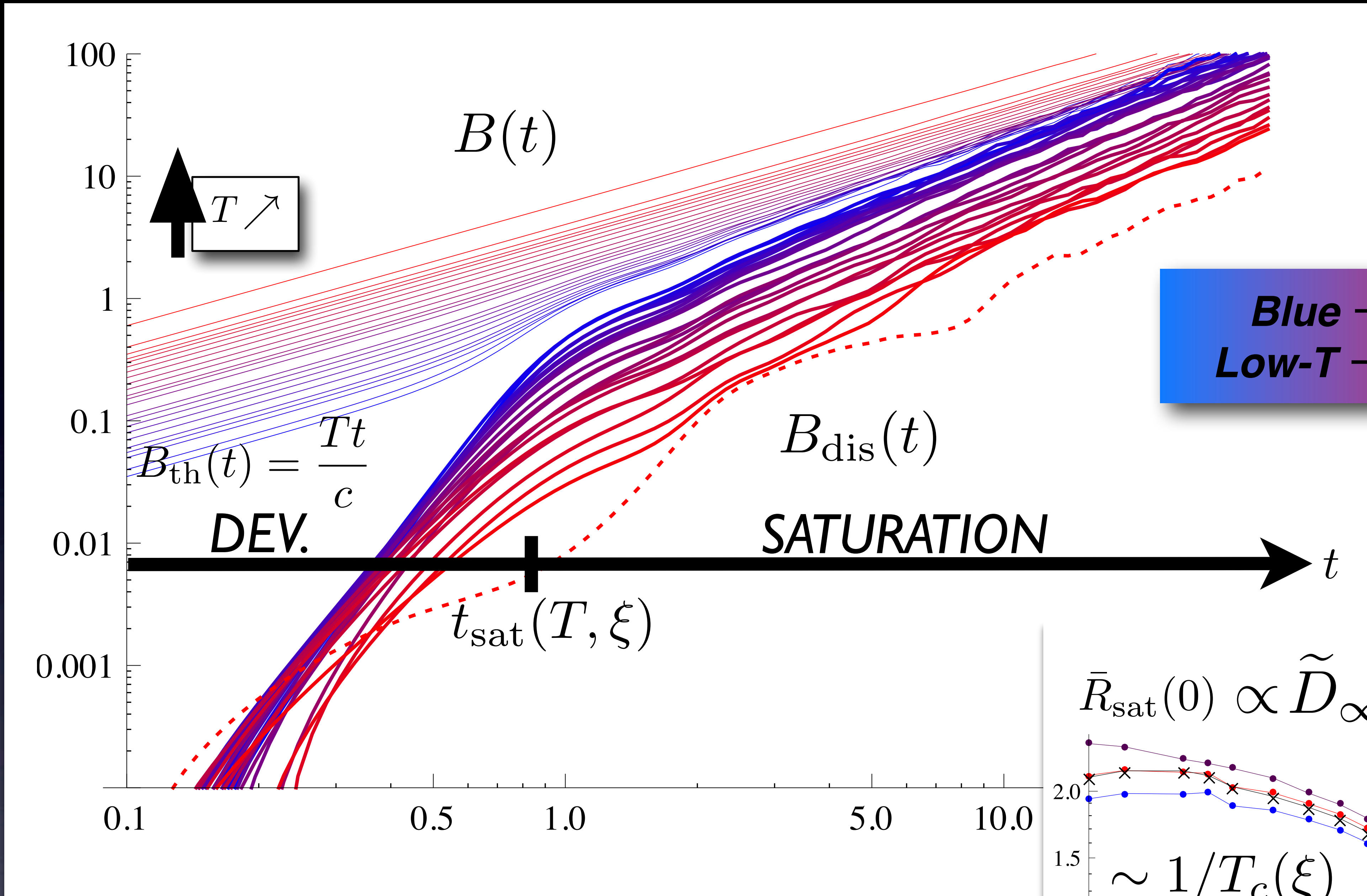




$$B(t) \equiv \overline{\langle y(t)^2 \rangle} = B_{\rm thermal}(t) + B_{\rm dis}(t)$$

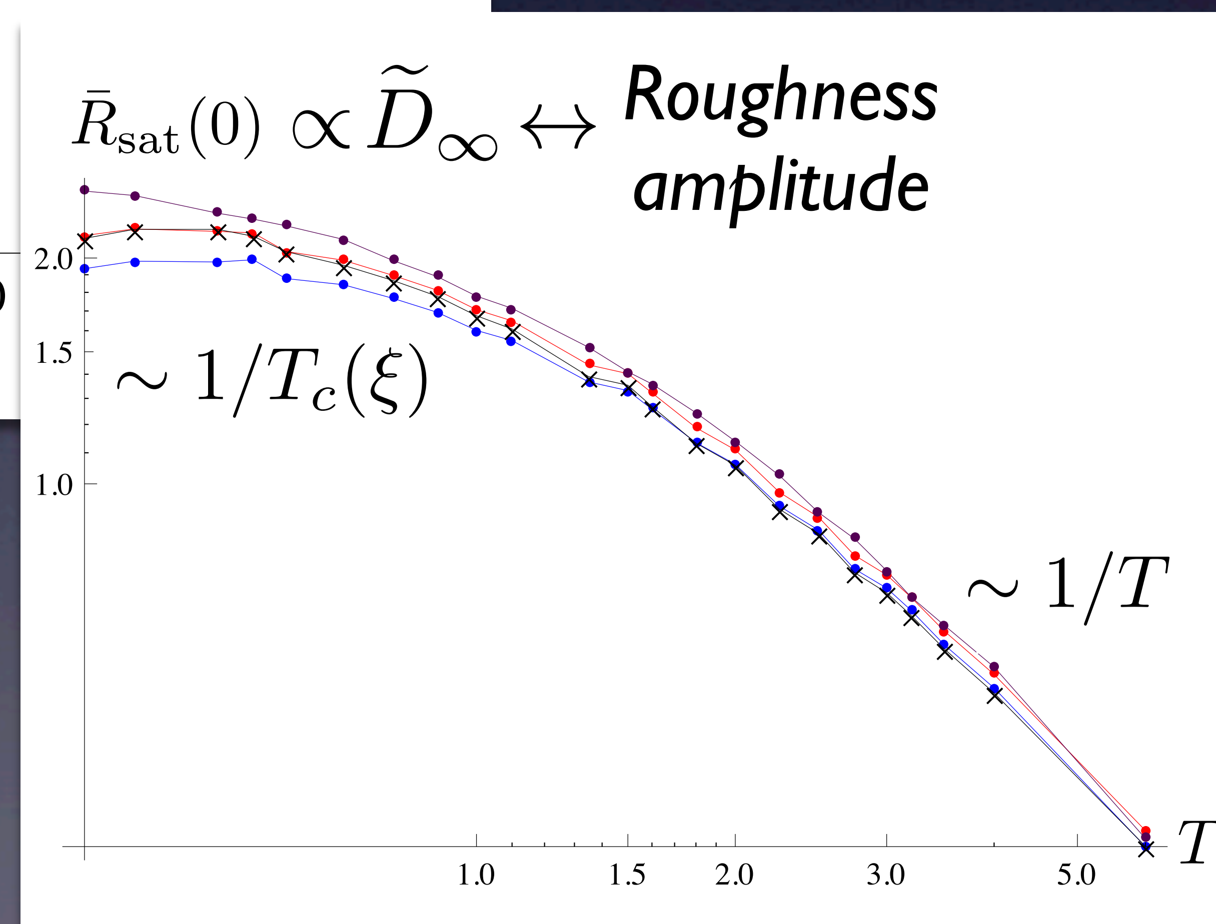
$$\begin{cases} B(t) \overset{(t\to\infty)}{\sim} A_{(c,D,T,\xi)}\, t^{4/3} \\ A_{(c,D,T,\xi)} \sim (\widetilde{D}_\infty / c^2)^{2/3} \end{cases}$$

Numerics: temperature dependence of the roughness $(\xi > 0)$



$$T_c \equiv (\xi c D)^{1/3}$$

Blue → Red: Low-T → High-T

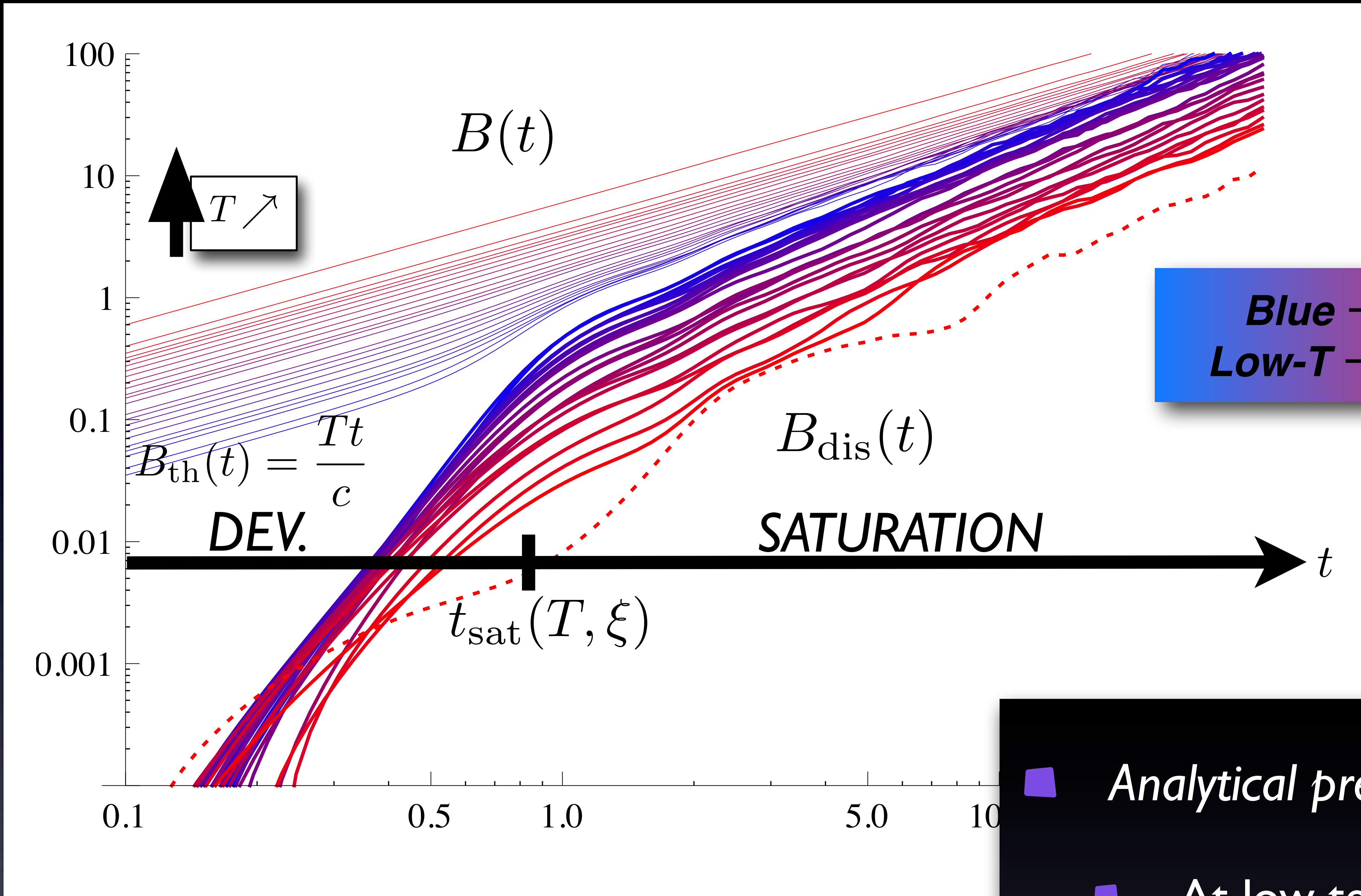


$$B(t) \equiv \overline{\langle y(t)^2 \rangle} = B_{\text{thermal}}(t) + B_{\text{dis}}(t)$$

$$\begin{cases} B(t) \overset{(t\to\infty)}{\sim} A_{(c,D,T,\xi)}\, t^{4/3} \\ A_{(c,D,T,\xi)} \sim (\widetilde{D}_\infty / c^2)^{2/3} \end{cases}$$

Numerics: temperature-dependence of the roughness $(\xi > 0)$

$$T_c \equiv (\xi c D)^{1/3}$$



Blue → Red: Low-T → High-T

$$B(t) \equiv \overline{\langle y(t)^2 \rangle} = B_{\mathrm{thermal}}(t) + B_{\mathrm{dis}}(t)$$

$$\begin{cases} B(t) \overset{(t\to\infty)}{\sim} A_{(c,D,T,\xi)}\, t^{4/3} \\ A_{(c,D,T,\xi)} \sim (\widetilde{D}_\infty / c^2)^{2/3} \end{cases}$$

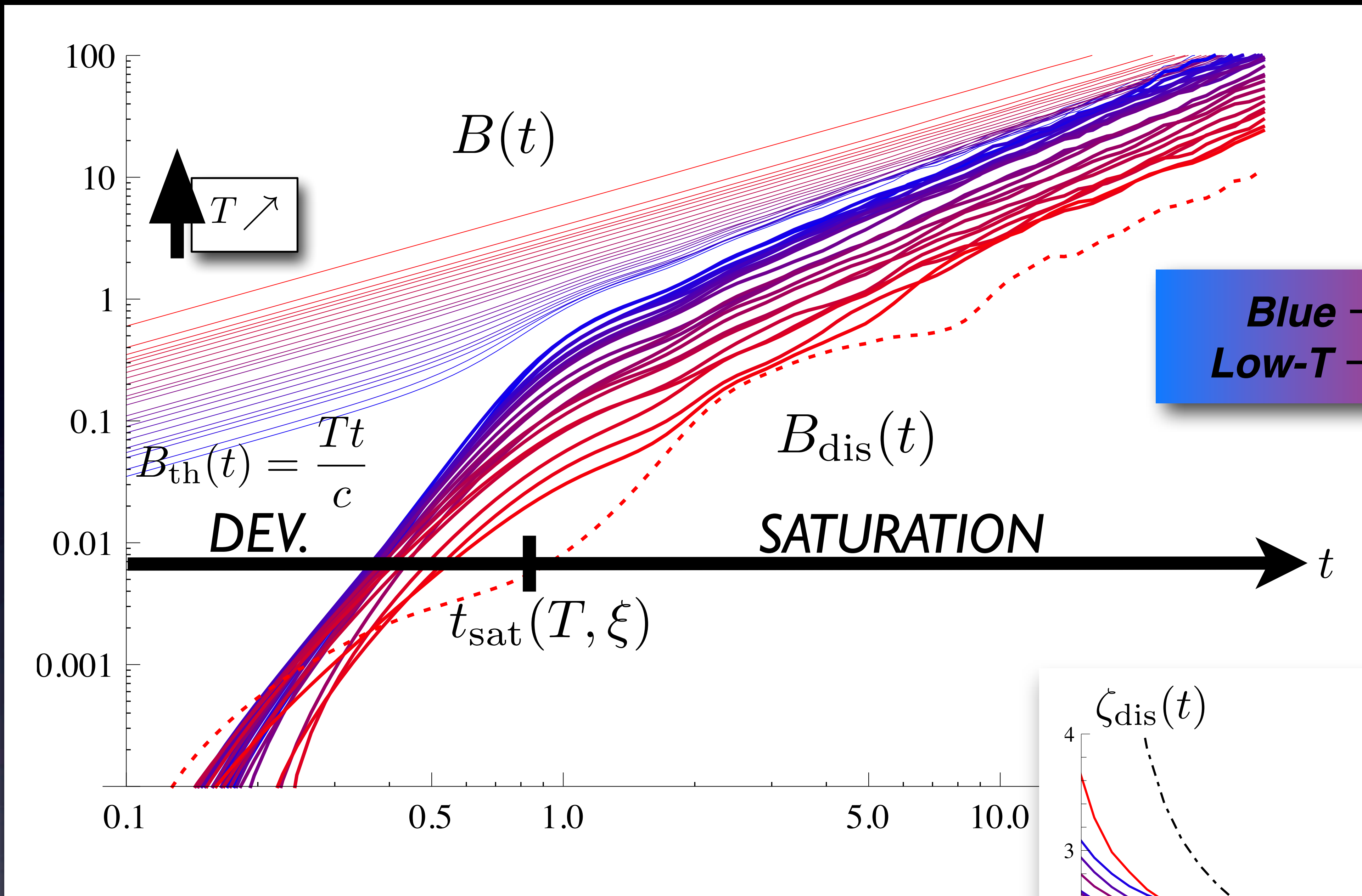
- *Analytical predictions:*
  - At low temperature: $T \ll T_c$

    $$A_{(c,D,0,\xi)} \sim \xi^{-2/9} T^0$$

  - At high temperature: $T \gg T_c$

    $$A_{(c,D,T,0)} \sim T^{-2/3} \xi^0$$

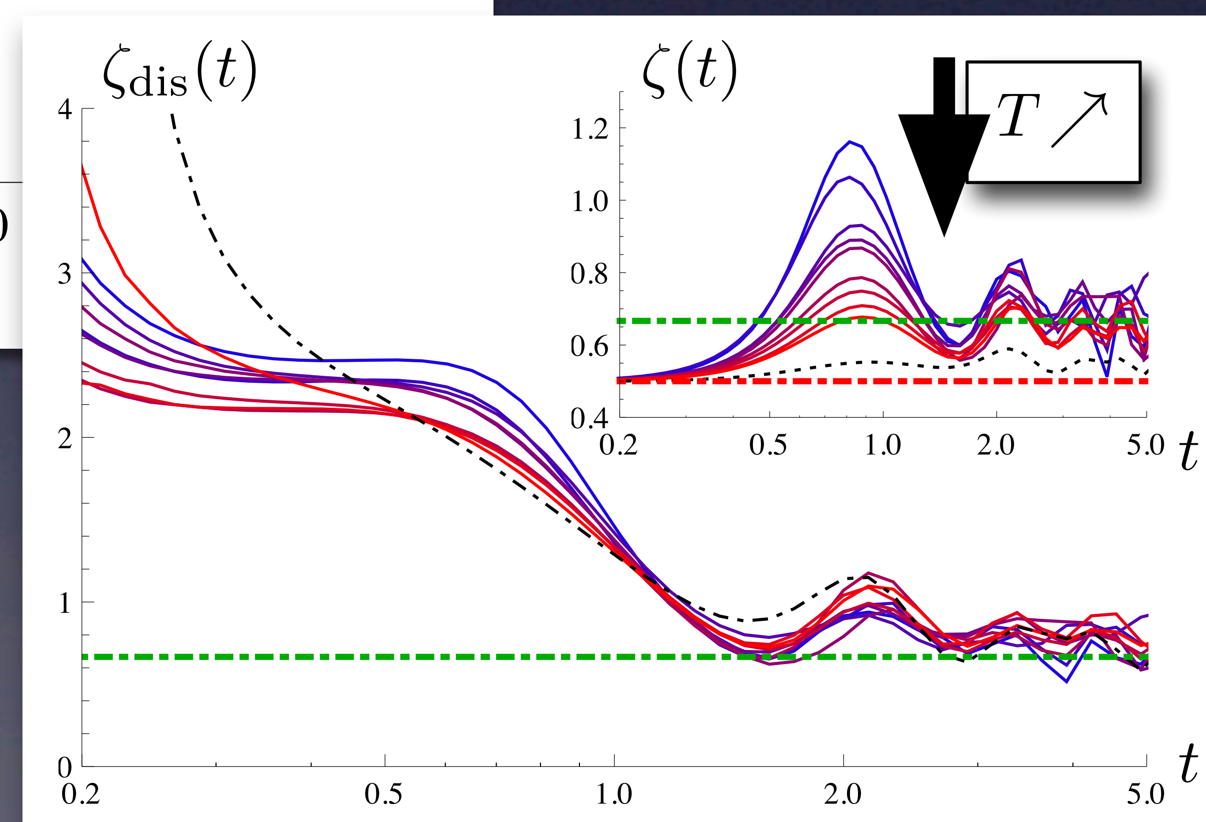
Numerics: temperature-dependence of the roughness $(\xi > 0)$



$$T_c \equiv (\xi c D)^{1/3}$$

Blue → Red:
Low-T → High-T

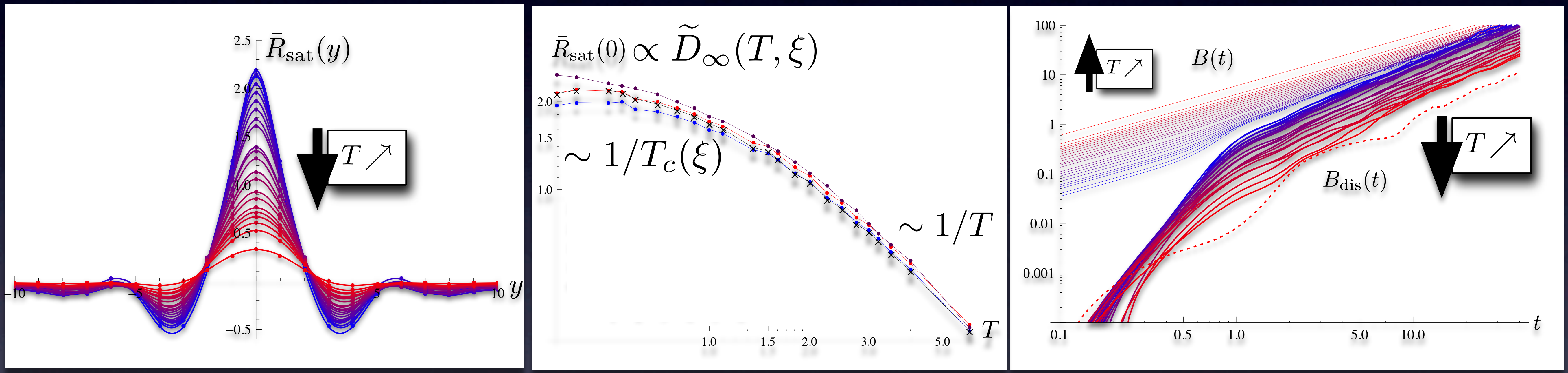
$$\zeta_{(\rm dis)}(t) = \frac{\partial \log B_{(\rm dis)}(t)}{\partial \log t}$$



$$B(t) \equiv \overline{\langle y(t)^2 \rangle} = B_{\rm thermal}(t) + B_{\rm dis}(t)$$

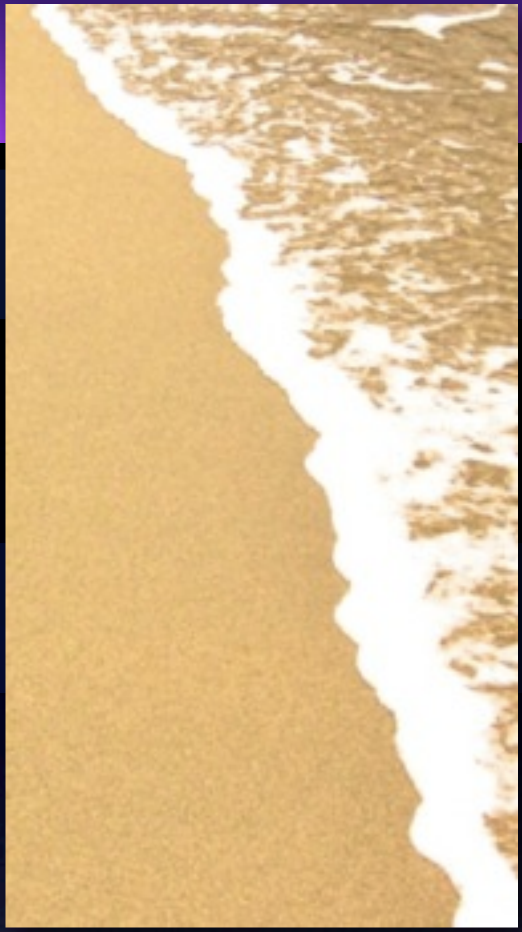
Summary

- Study of the interplay between finite temperature & finite width/disorder correlation length ξ
- Effective description at fixed lengthscale: fluctuations of DP free-energy at fixed 'time'



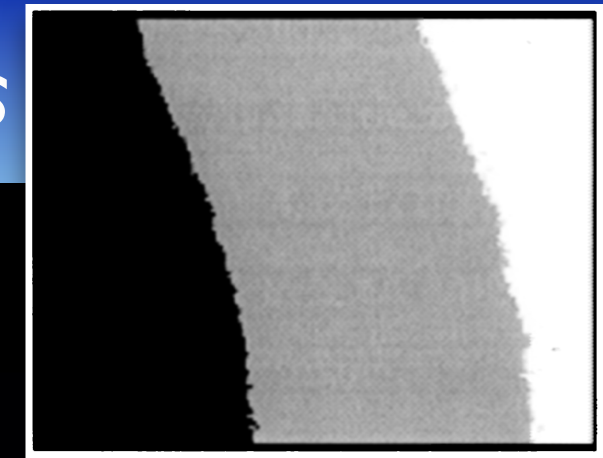
- Regimes in the disorder free-energy fluctuations & roughness
- Crossover in temperature controlled by free-energy amplitude $\widetilde{D}_\infty(T,\xi)$ & characteristic temperature $T_c(\xi) = (\xi c D)^{1/3}$
- Imprint of the microscopic disorder correlator in free-energy correlator $\bar{R}_{\rm sat}(y)$

Outline



- Introduction
  - Generic framework: Disordered Elastic Systems (DES)
  - Specific issue: role a finite width or disorder correlation length
- Model of a one-dimensional interface
  - Geometrical fluctuations and roughness
  - DES model of a one-dimensional (1D) interface
  - Static 1D interface versus 1+1 Directed Polymer (DP)
- Temperature-dependent fluctuations
  - Disorder free-energy fluctuations
  - Roughness: temperature-induced crossover
- Link to prototypal experiments
  - Ferromagnetic domain walls in ultrathin films
  - Growing interfaces in nematic liquid crystals
- Perspectives
  - 1D Kardar-Parisi-Zhang (KPZ) universality class

Experimental 1D interfaces: magnetic domain walls



Domain walls in ultrathin Pt/Co/Pt ferromagnetic films

- Static & Dynamical study:

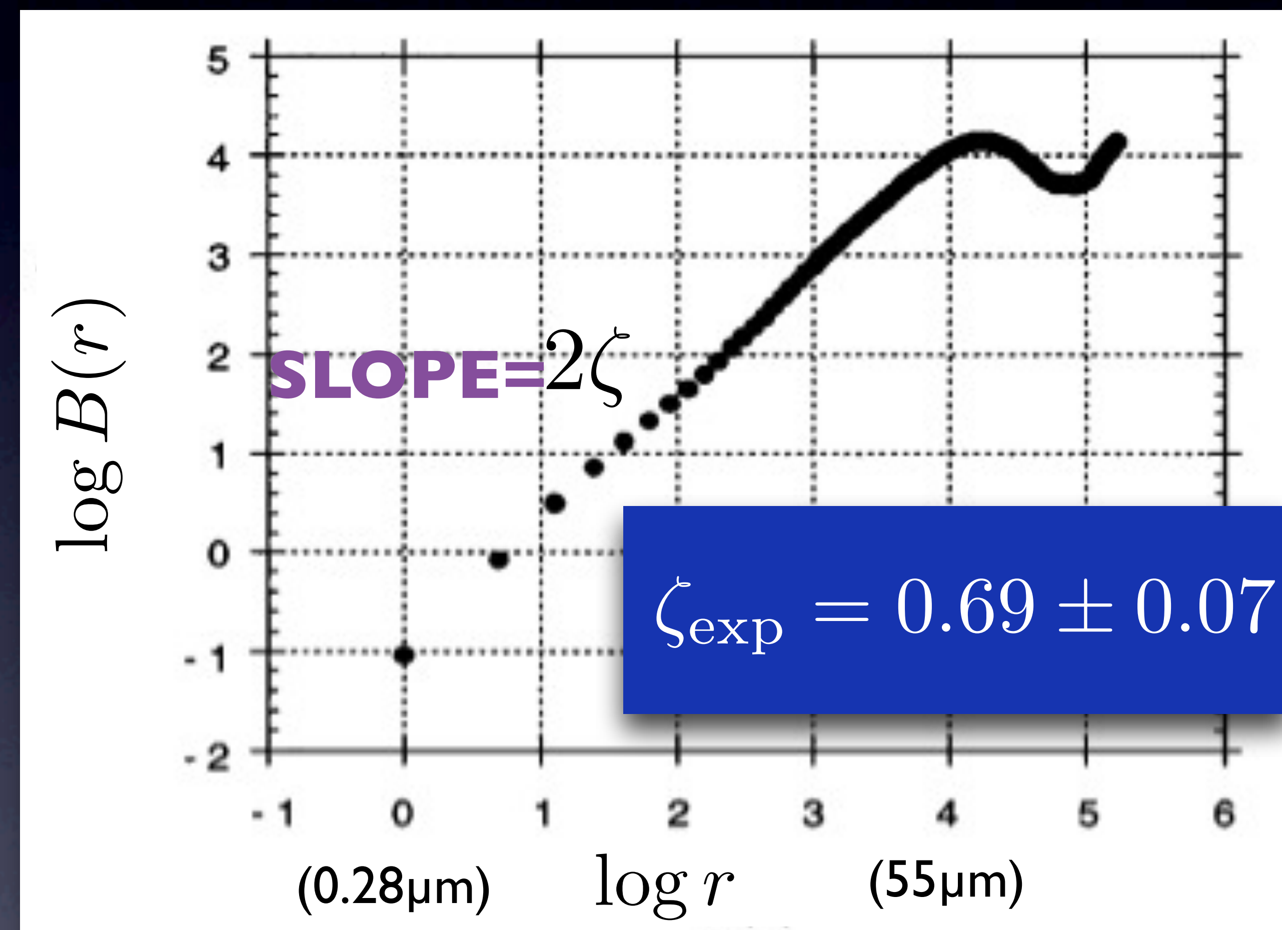
  Prototypal experimental realization of a 1D interface with short-range elasticity & random-bond quenched disorder

- Our prediction for the static roughness amplitude: $B(r) \sim A_{(c,D,T,\xi)}\, r^{2\zeta}$

$$\begin{cases} \zeta_{\mathrm{RM}} = 2/3 \\ A_{(c,D,T,\xi)} \sim (\widetilde{D}_{\infty}/c^2)^{2/3} \end{cases}$$

- Temperature dependence of the model effective parameters?

- Already at 'low temperature' when at room temperature?



S. Lemerle et al., *Phys. Rev. Lett.* **80**, 849 (1998).

P. J. Metaxas et al., *Phys. Rev. Lett.* **99**, 217208 (2007).

V. Repain et al., *Europhys. Lett.* **68**, 460 (2004).

S. Bustingorry et al., *Phys. Rev. B* **85**, 214416 (2012).

Experimental 1D KPZ interfaces: nematic liquid crystals

$$\partial_t h(t,x) = \underbrace{\nu \nabla_x^2 h(t,x)} + \underbrace{\frac{\lambda}{2} \left[\nabla_x h(t,x)\right]^2} + \underbrace{\eta(t,x)}$$

PRL **104,** 230601 (2010) PHYSICAL REVIEW LETTERS week ending 11 JUNE 2010

Universal Fluctuations of Growing Interfaces: Evidence in Turbulent Liquid Crystals

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(Received 28 January 2010; published 11 June 2010)

K. Takeuchi & M. Sano, *Phys. Rev. Lett.* **104**, 230601 (2010).

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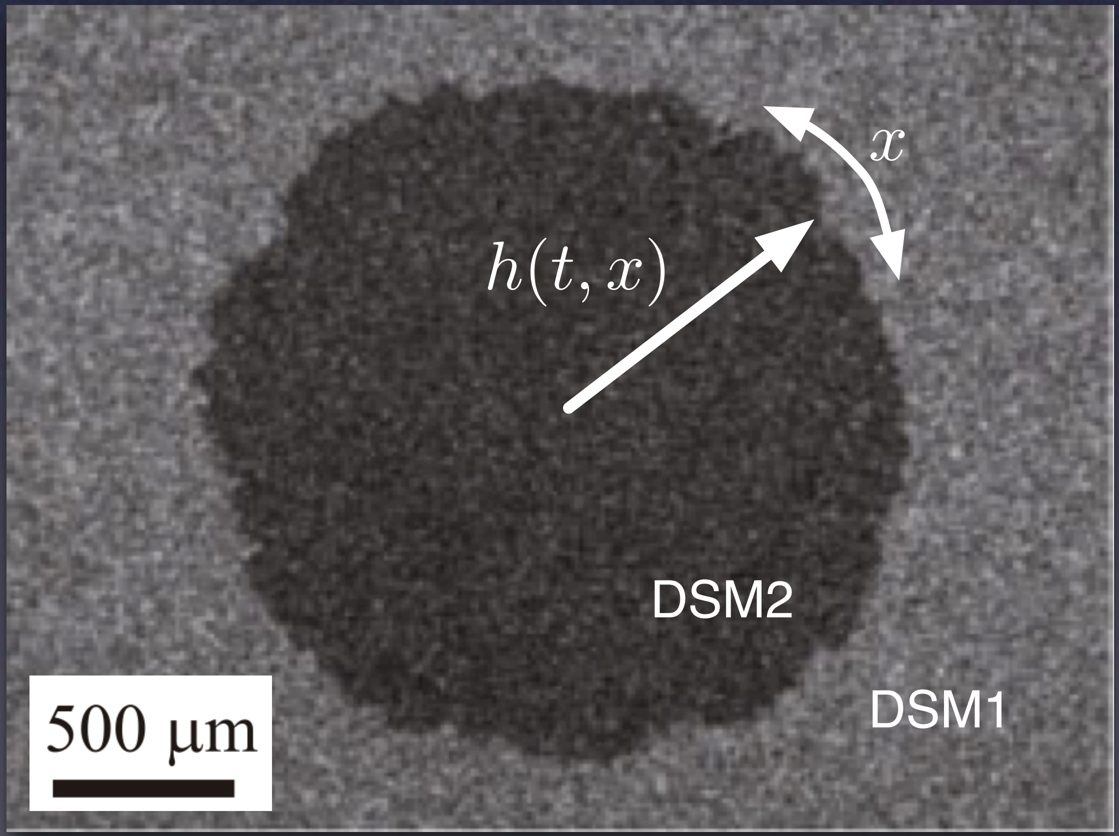

Experimental 1D KPZ interfaces: nematic liquid crystals

$$\partial_t h(t,x) = \underbrace{\nu \nabla_x^2 h(t,x)} + \underbrace{\frac{\lambda}{2}\left[\nabla_x h(t,x)\right]^2} + \underbrace{\eta(t,x)}$$

PRL 104, 230601 (2010) PHYSICAL REVIEW LETTERS week ending 11 JUNE 2010

Universal Fluctuations of Growing Interfaces: Evidence in Turbulent Liquid Crystals

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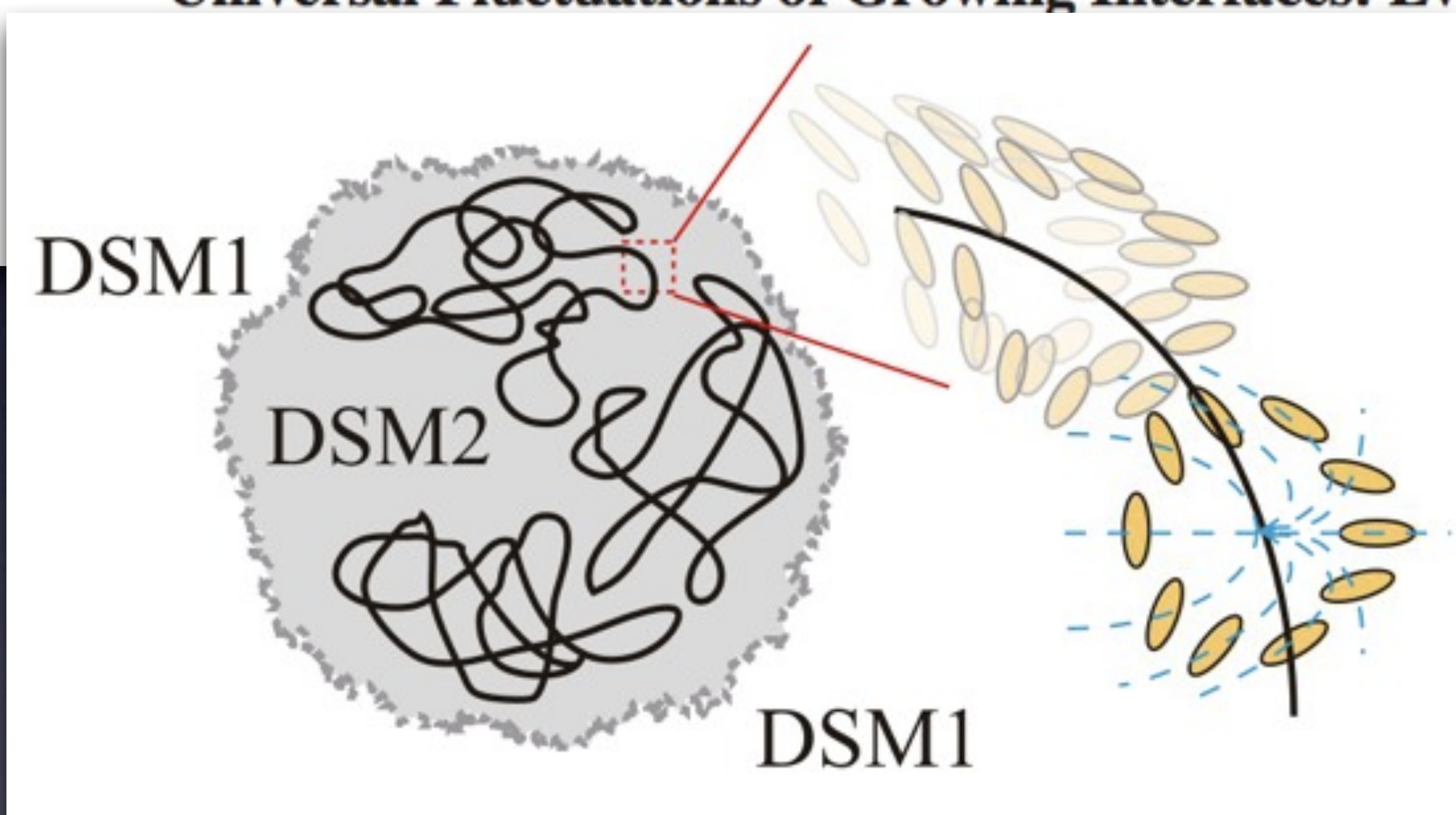
ed 11 June 2010)



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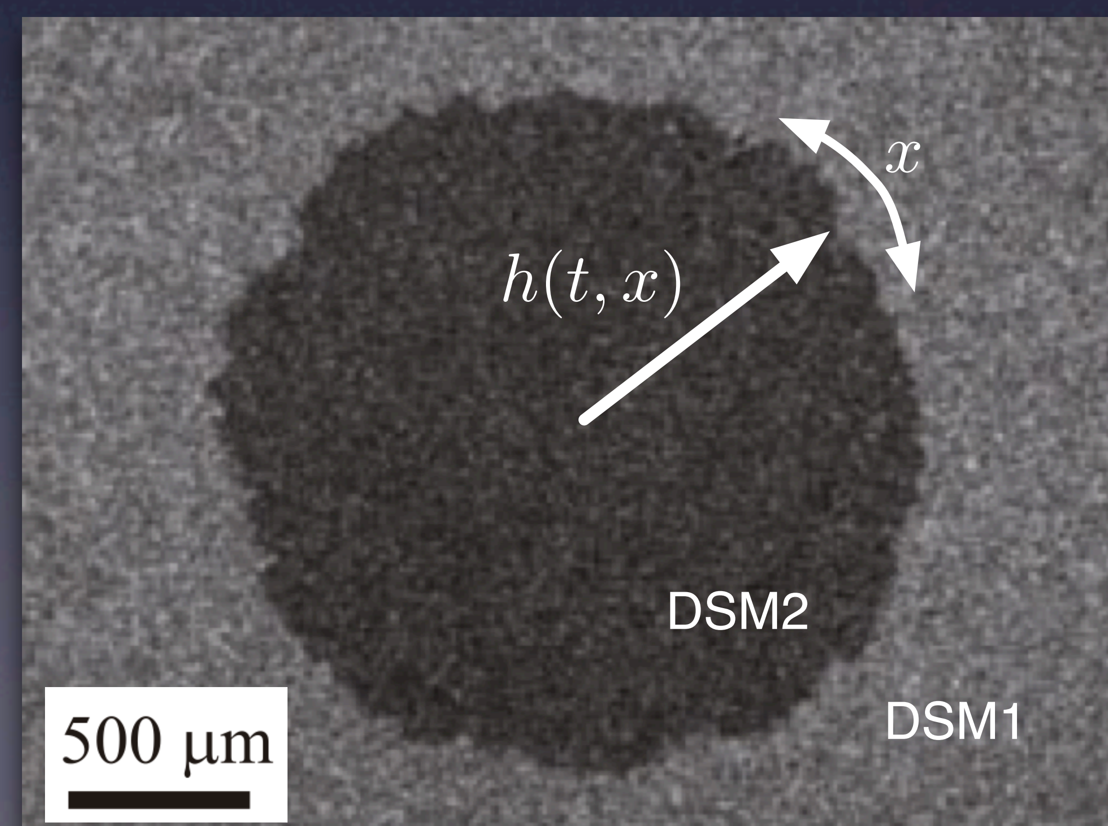

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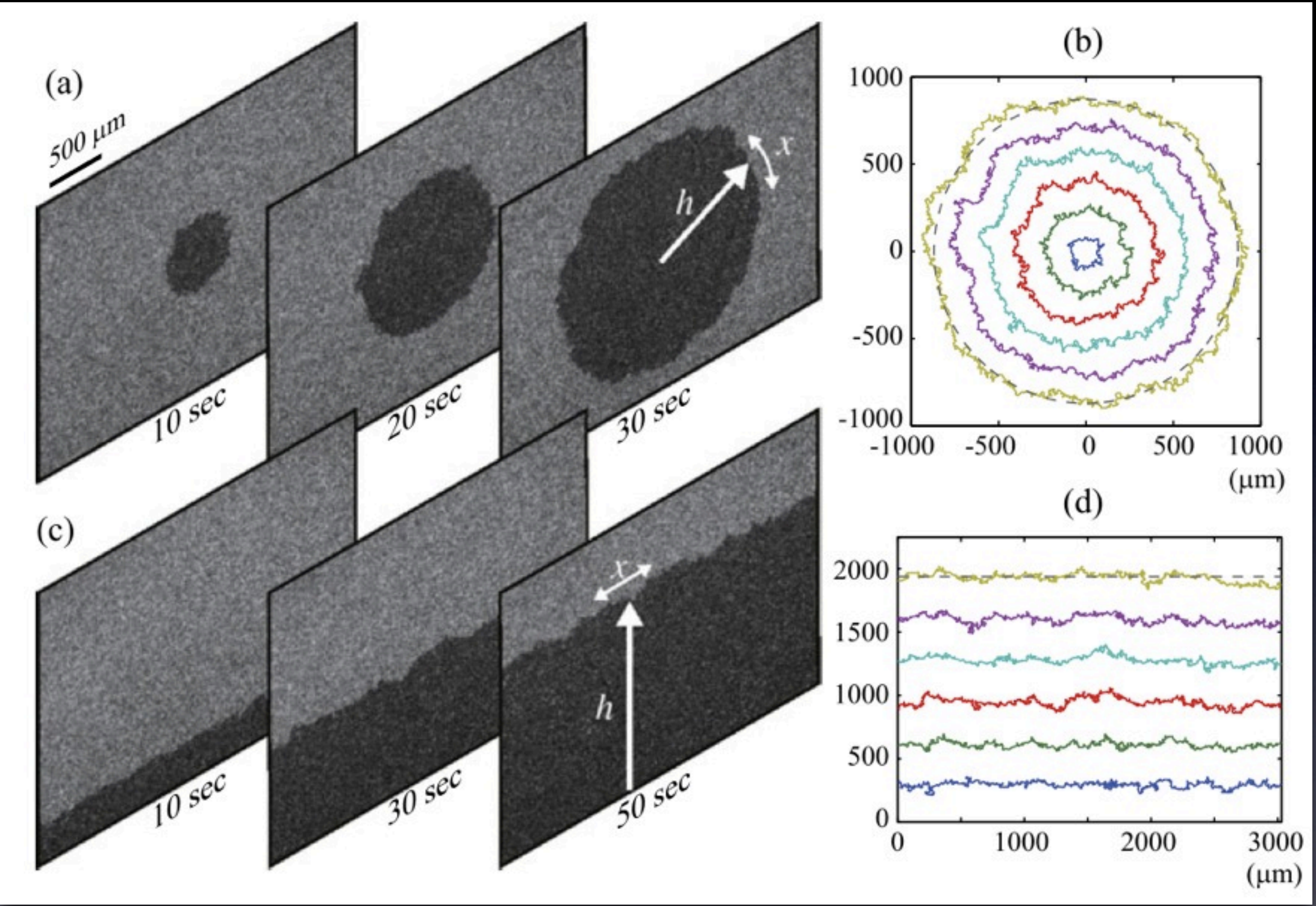

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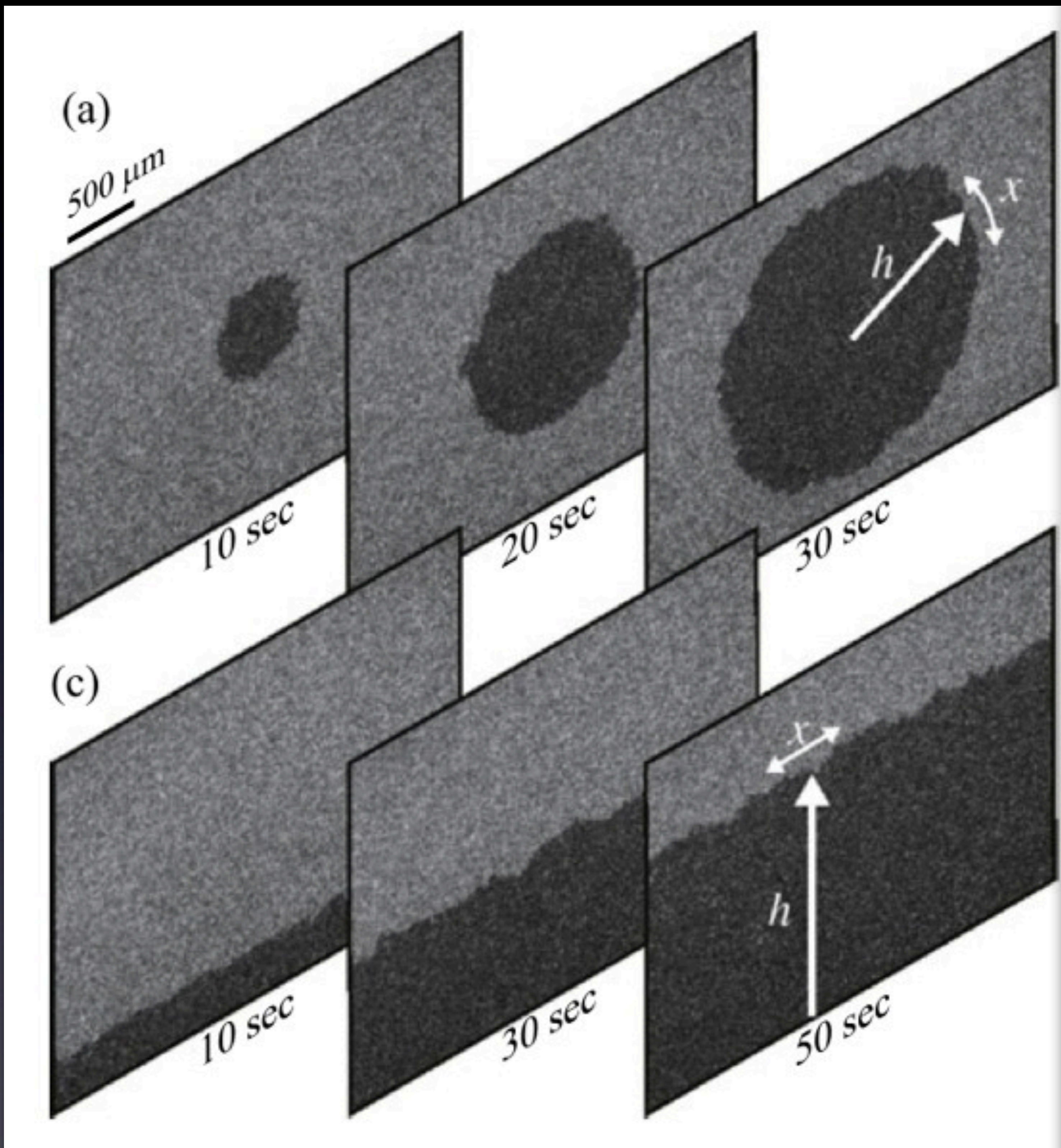
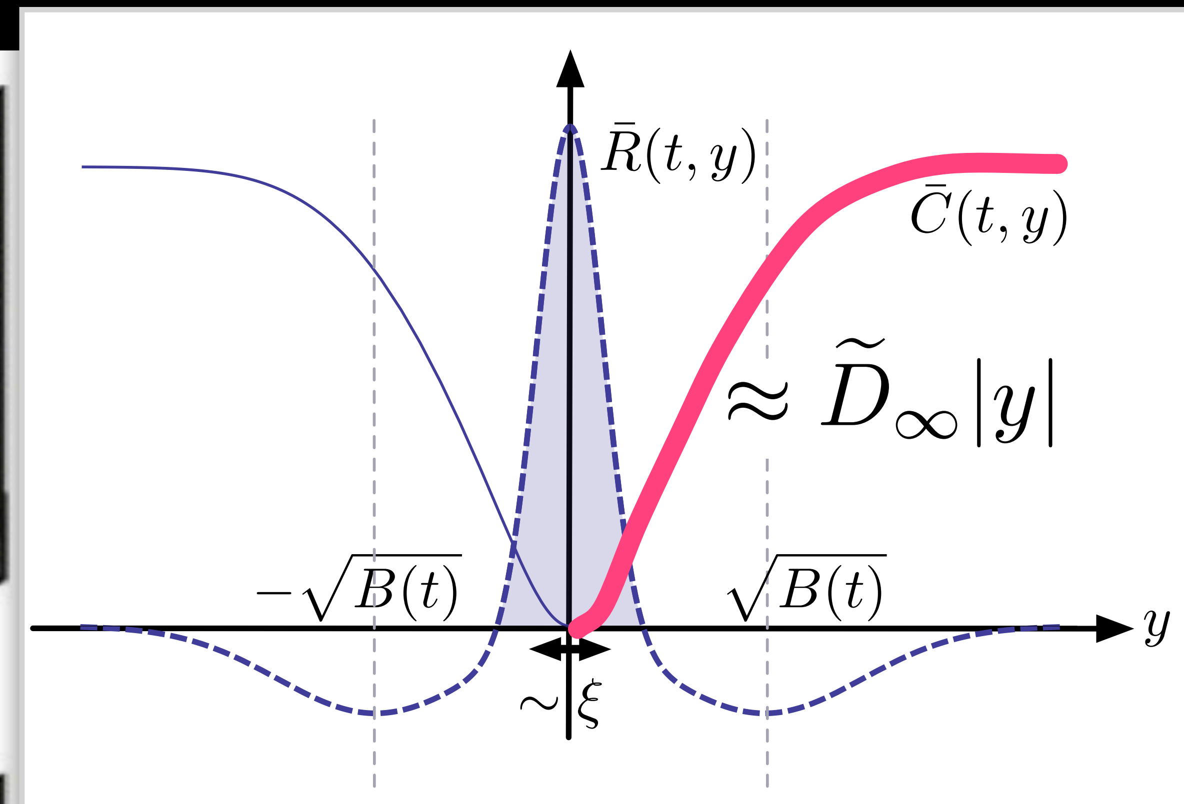
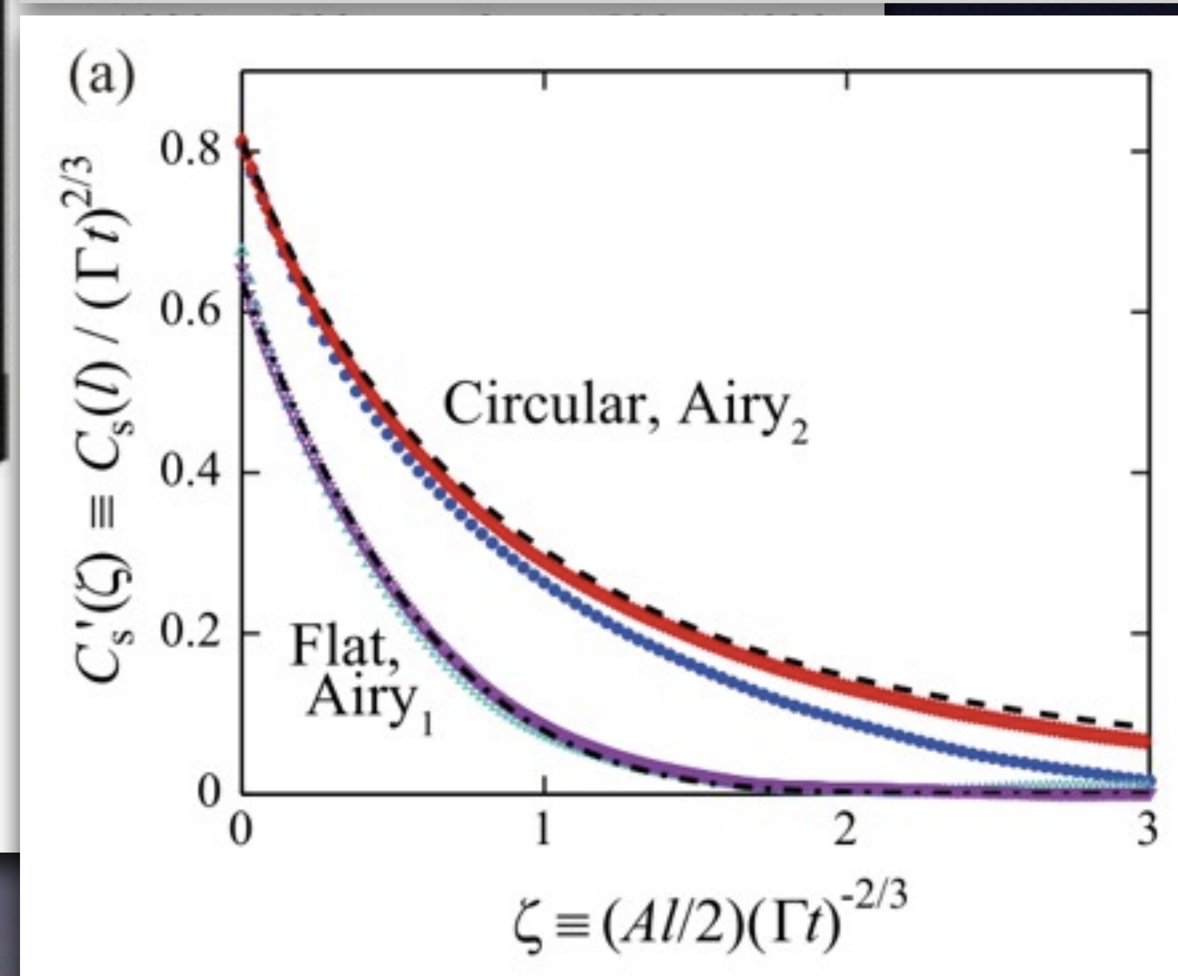
Experimental 1D KPZ interfaces: nematic liquid crystals





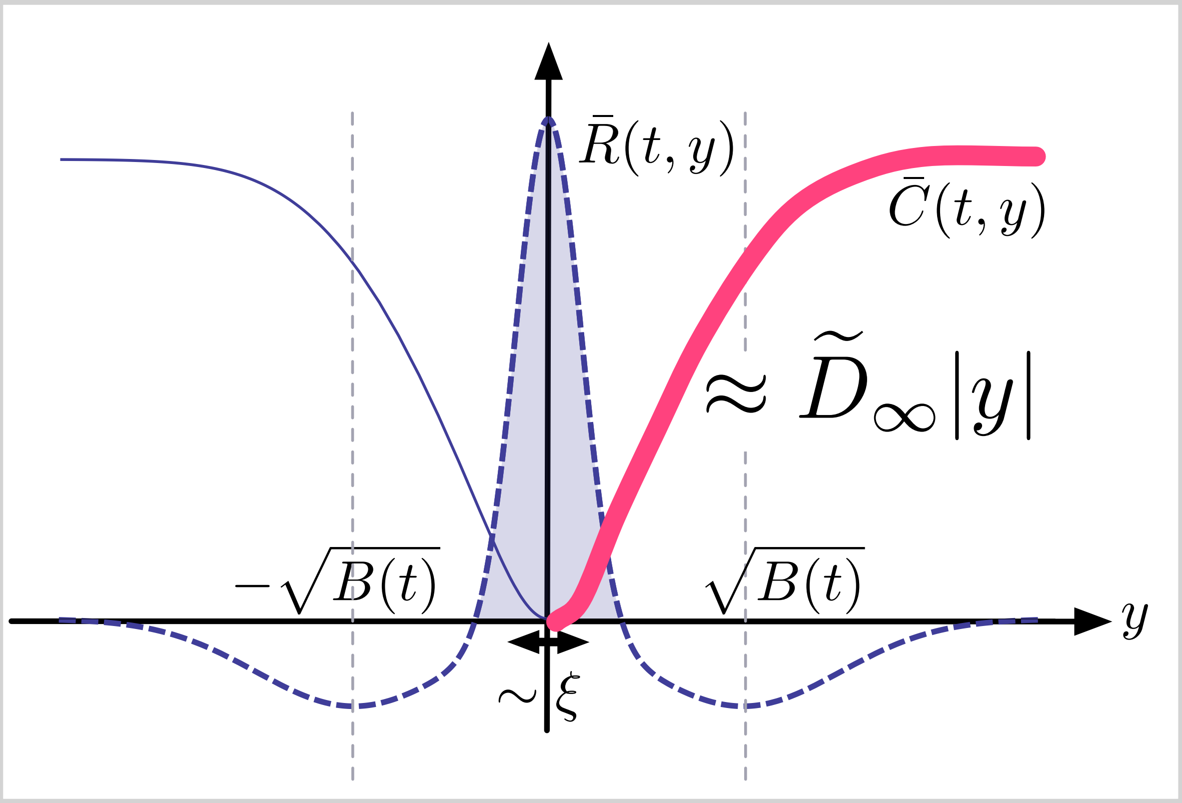


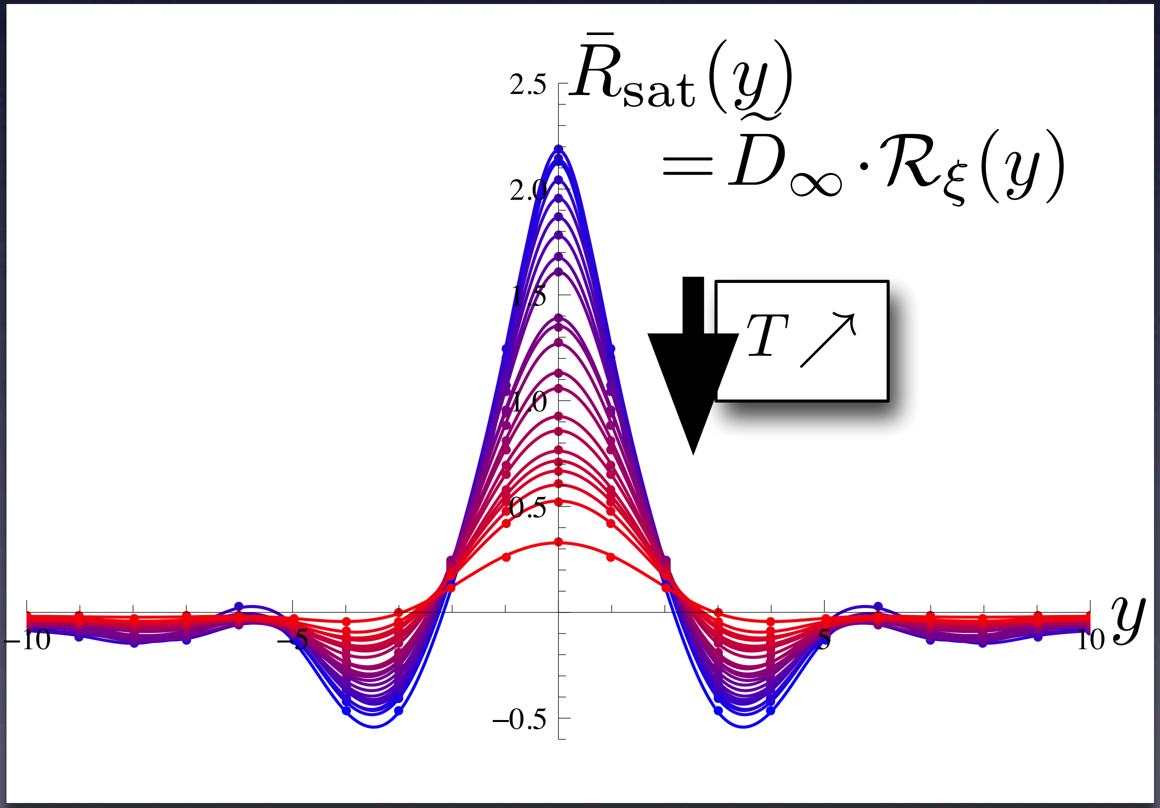
K. Takeuchi & M. Sano, *J. Stat. Phys.* **147**, 853 (2012).

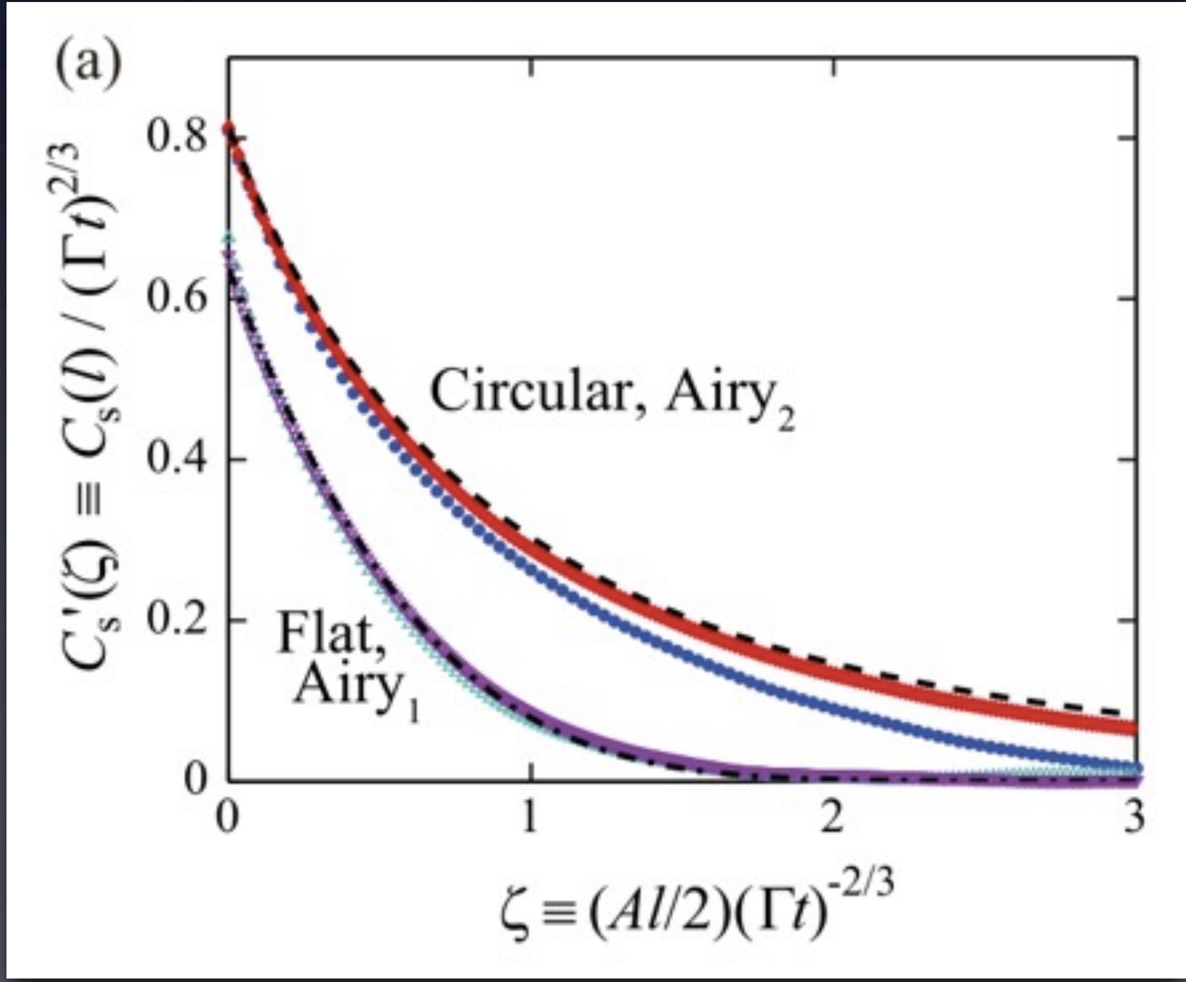
Experimental 1D KPZ interfaces: nematic liquid crystals

- Possible to observe the signature of a finite disorder correlation length?

- Shape of the asymptotic correlator accessible experimentally?



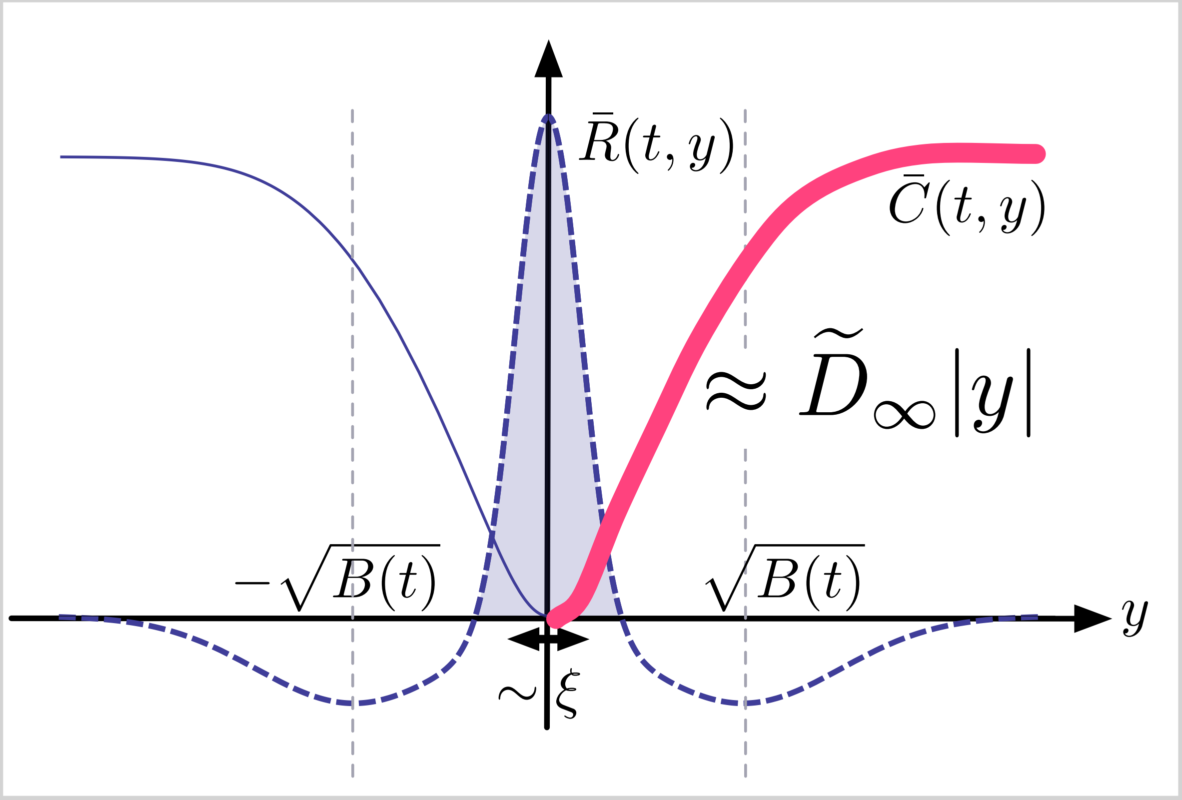


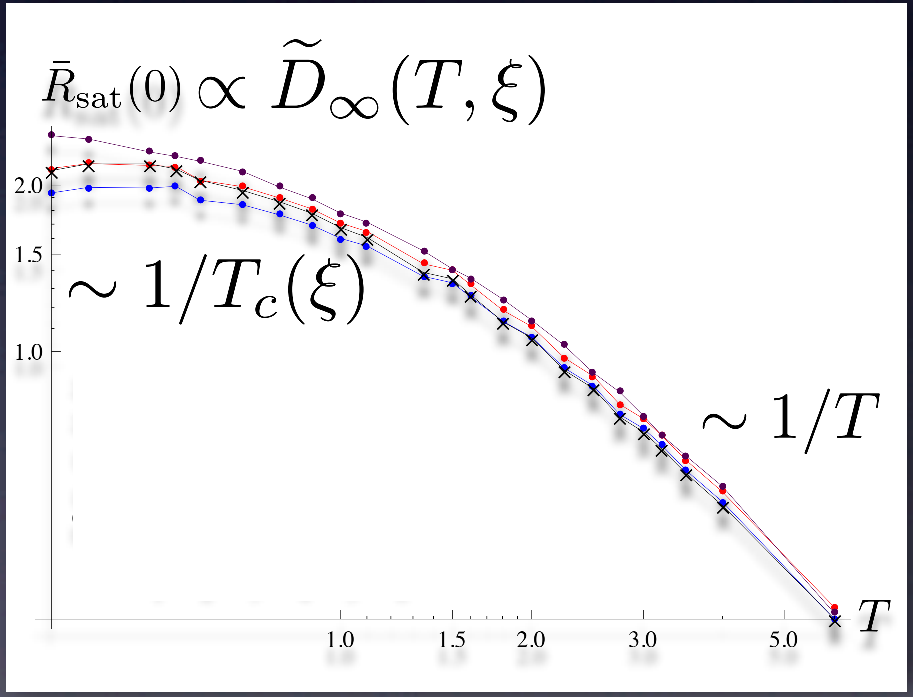


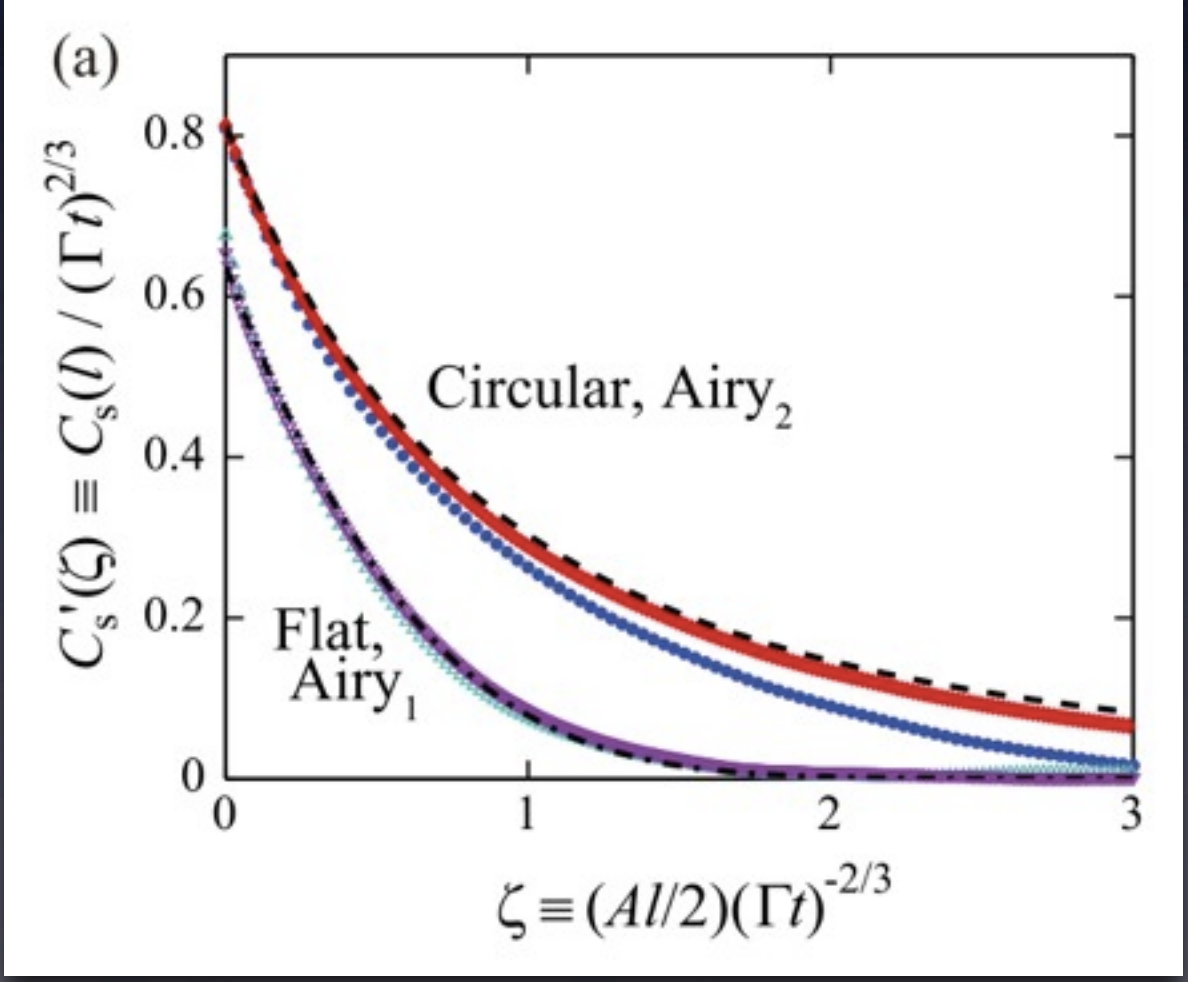
Experimental 1D KPZ interfaces: nematic liquid crystals

- Possible to observe the signature of a finite disorder correlation length?

- Is there a crossover of the amplitude of the asymptotic correlator?





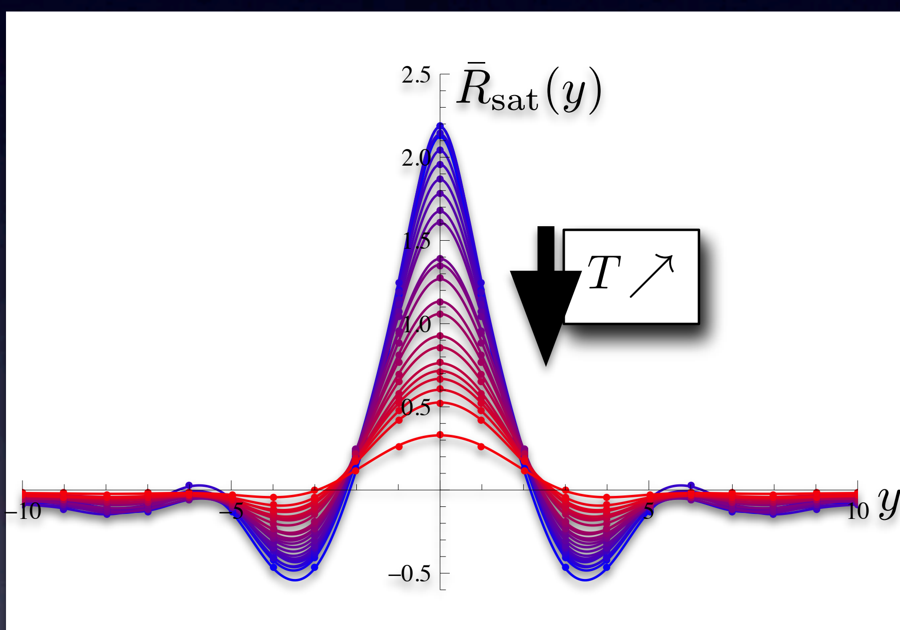


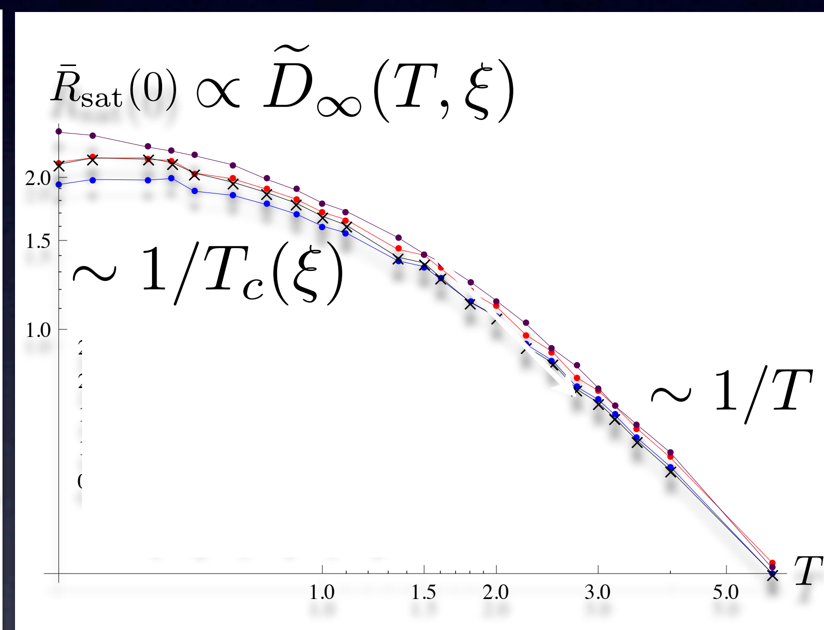
Low-temperature regime in our model

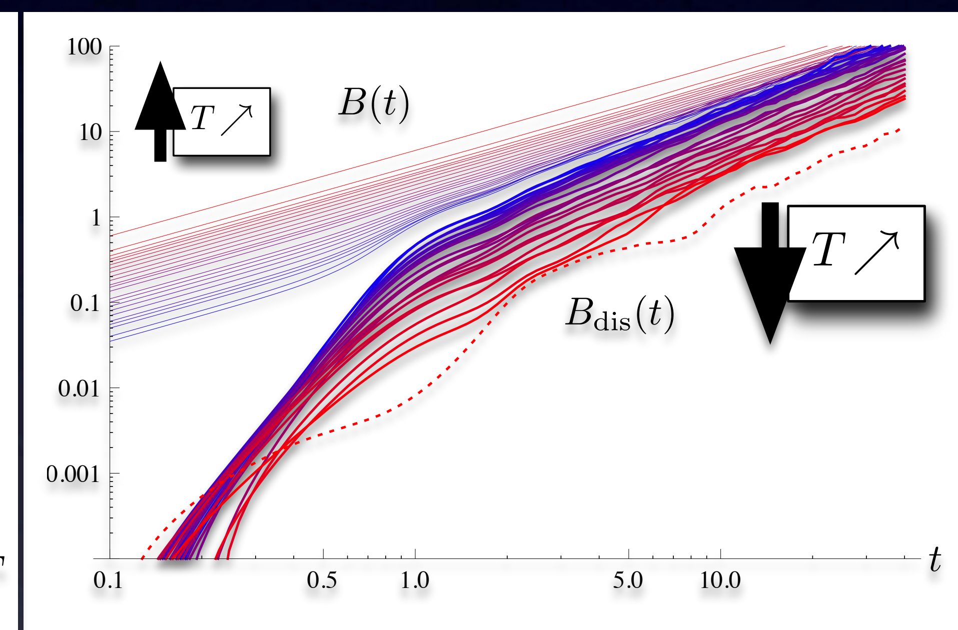
$\overset{?}{\Longleftrightarrow}$ High-velocity for the liquid crystals

Perspectives

- Theoretical predictions to be challenged with experimental interfaces:
  - Ferromagnetic domain walls? (low temperature)
  - Nematic liquid crystals? (high-velocity)
- Connections with the KPZ universality class: role of a correlated disorder/noise?

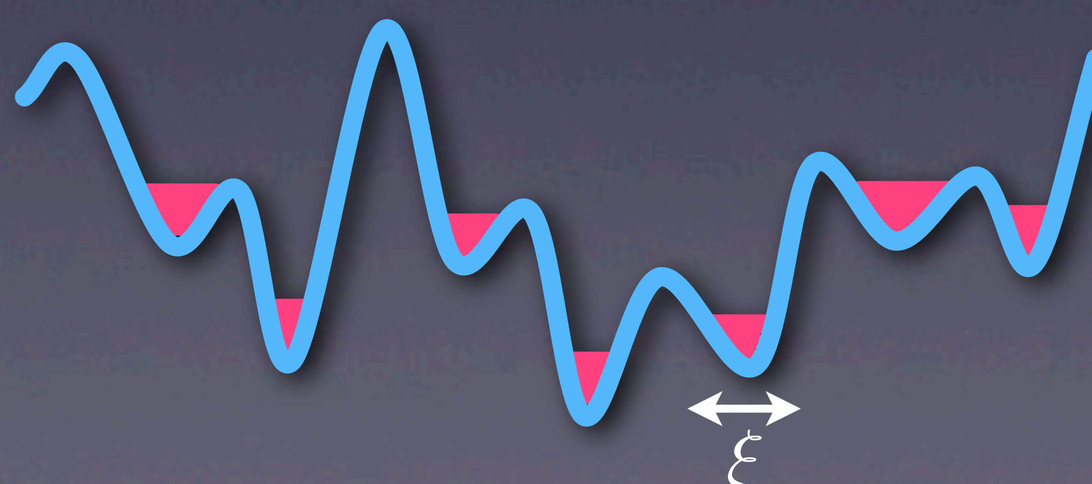






Low temperature

$$T \ll T_c(\xi) = (\xi c D)^{1/3}$$



Effective width

$$\xi \gg \xi_{\text{th}}(T) = \frac{T^3}{cD}$$

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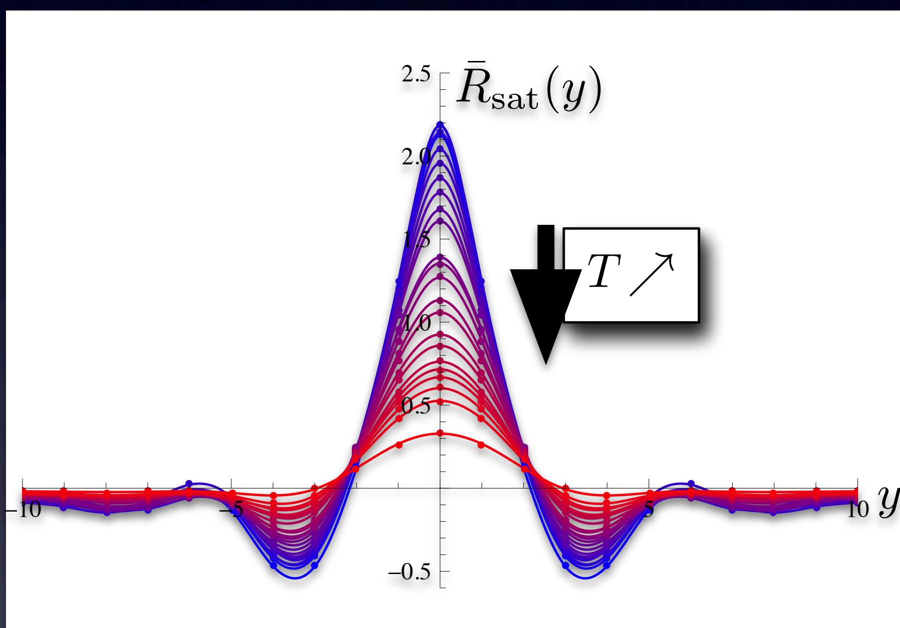

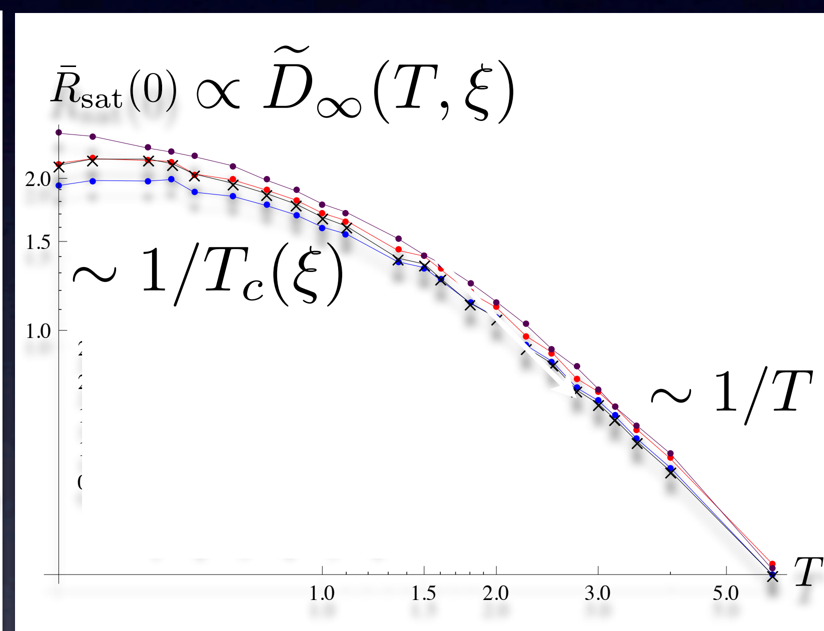

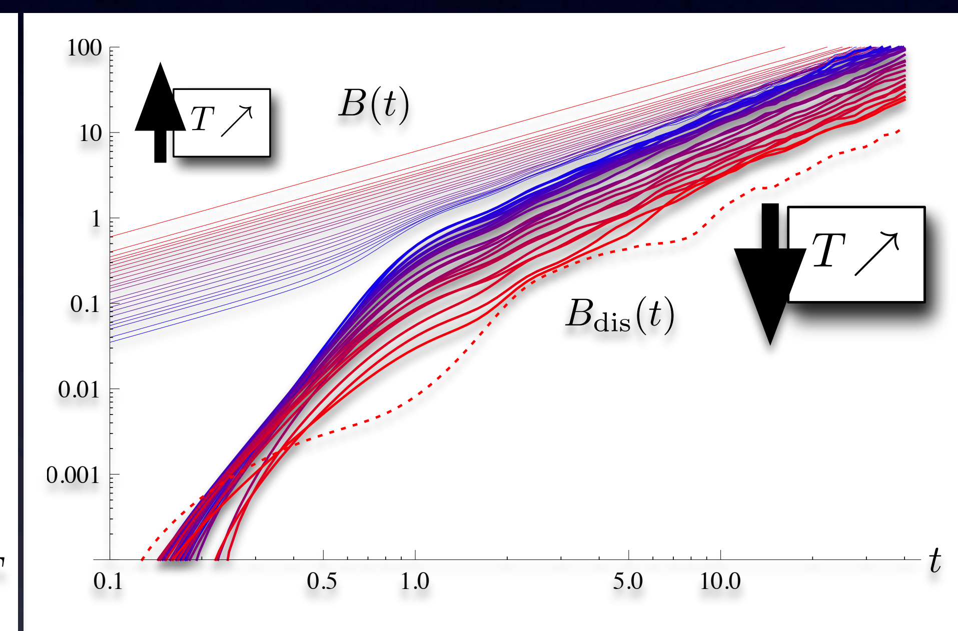

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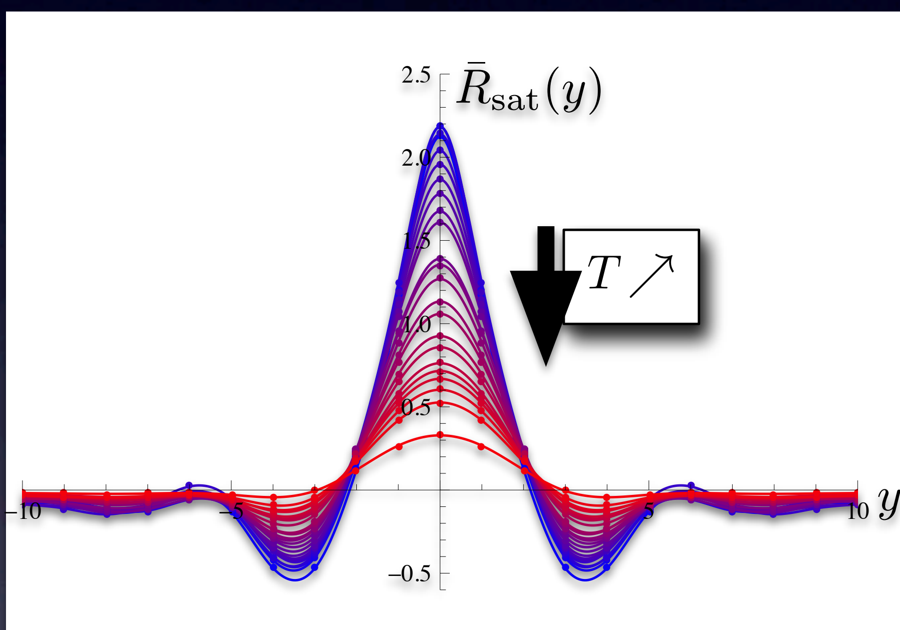
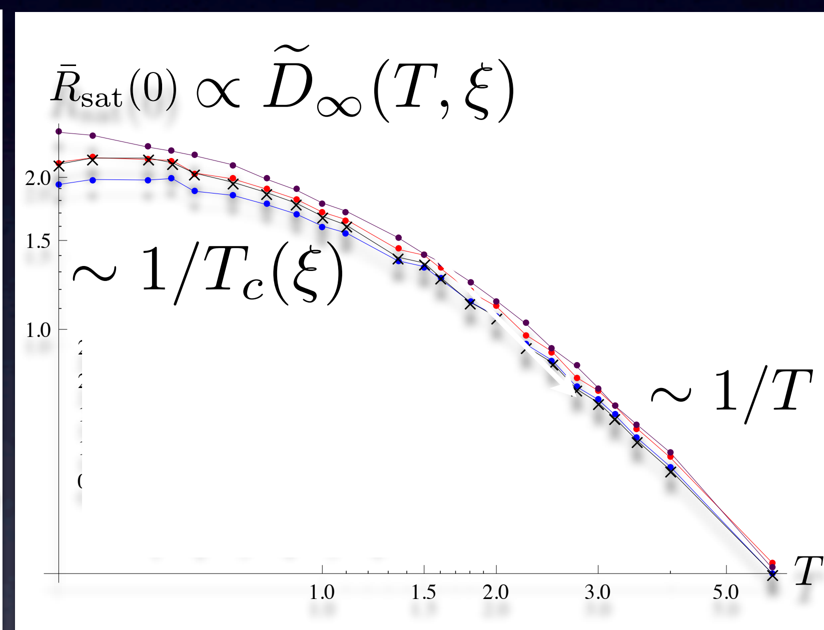
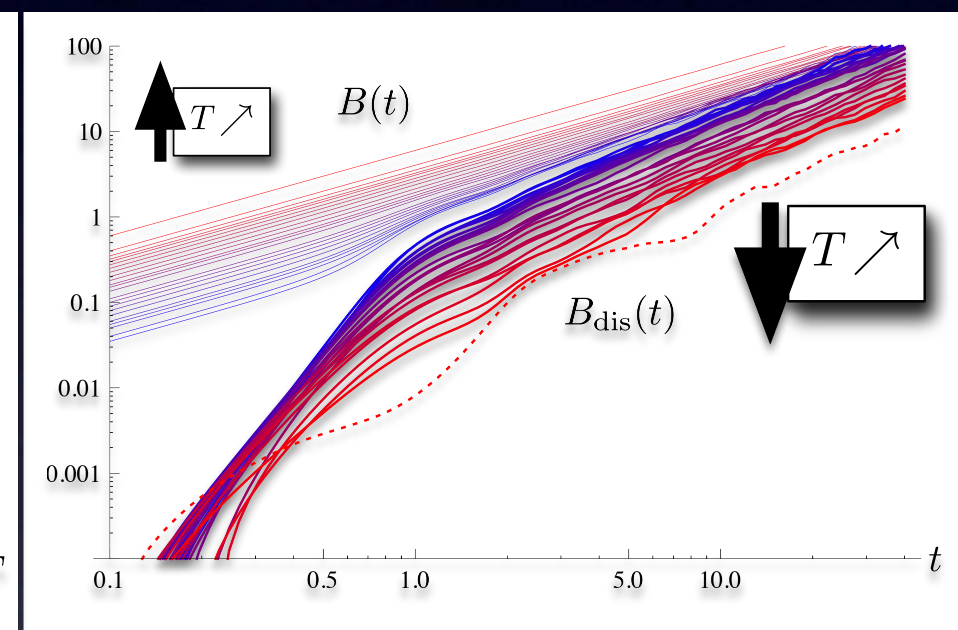


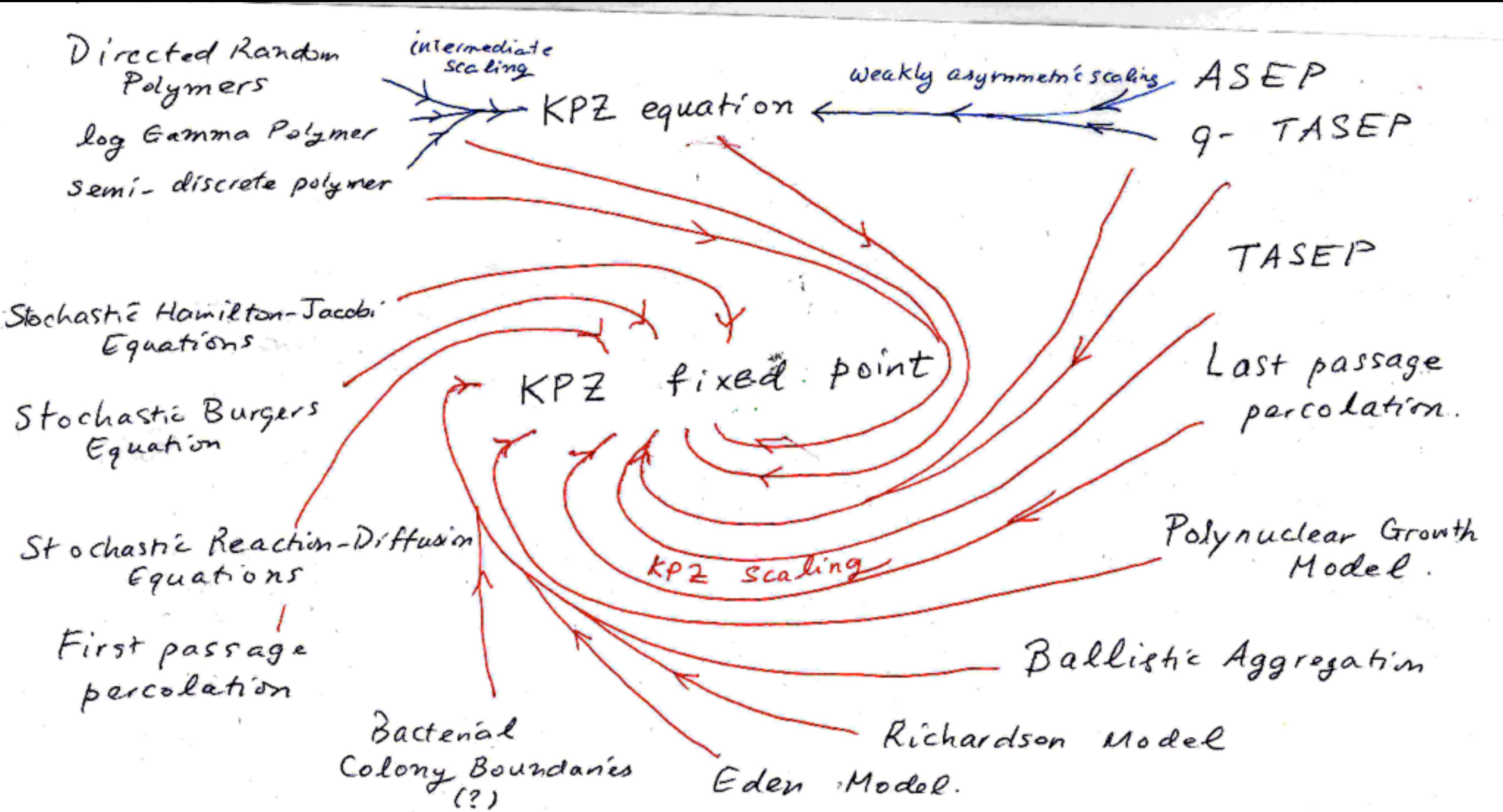




- Better understanding of the short-'time' regime hidden below t_{sat}

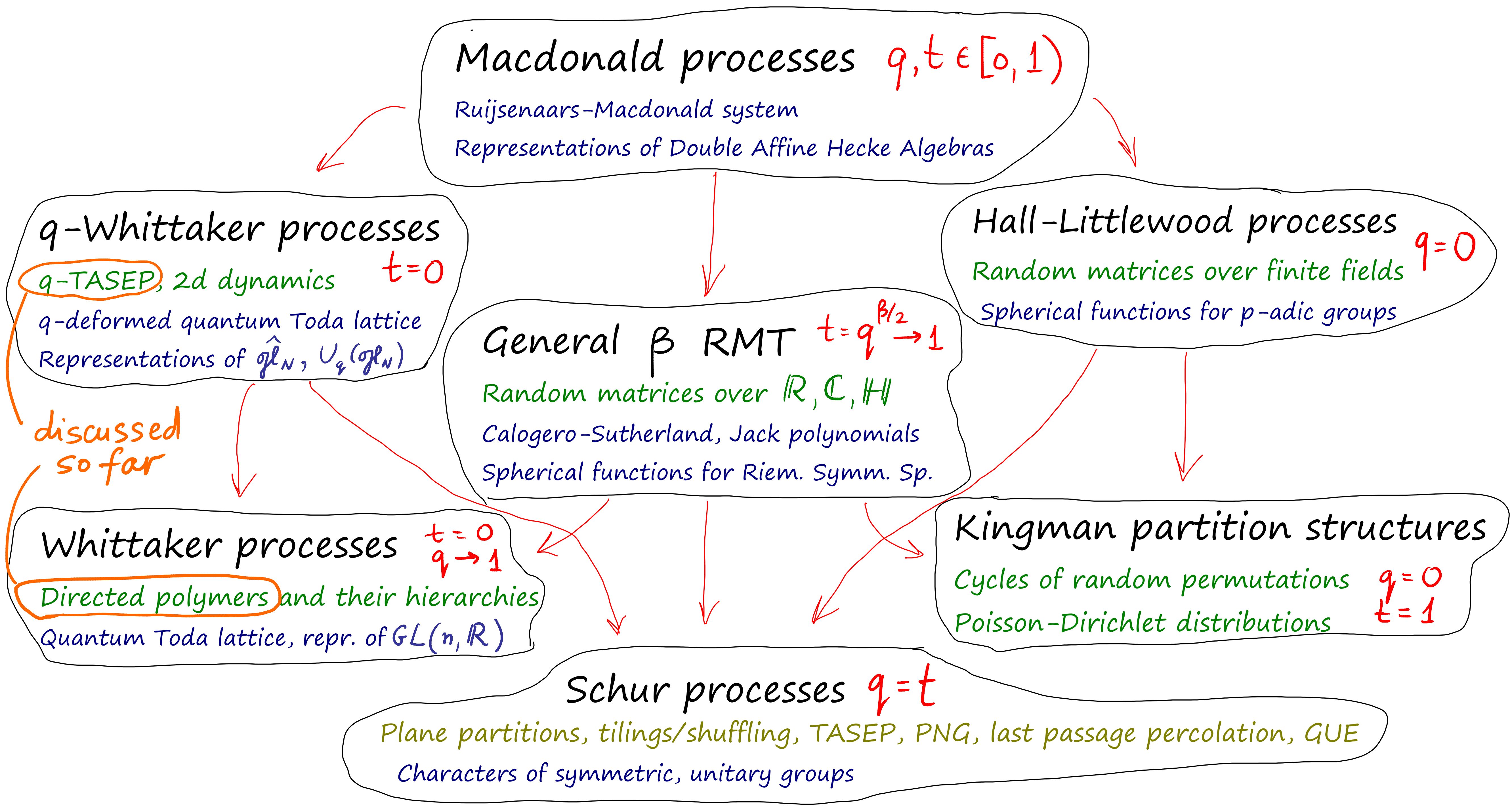
  & non-Gaussian fluctuations generated by the KPZ nonlinearity $\frac{1}{2c}\left[\partial_y \bar{F}_V(t,0)\right]^2$

Kardar-Parisi-Zhang (KPZ) universality class



- Cf. Talk by Jeremy Quastel, « The Kardar-Parisi-Zhang Equation and Universality Class» (2012), http://www.math.toronto.edu/quastel/talk.pdf.
- J. Quastel, « Introduction to KPZ », *CMD* 2011, http://www.math.toronto.edu/quastel/survey.pdf.

Kardar-Parisi-Zhang (KPZ) universality class



- Cf. Talk by Ivan Corwin, « Integrable probability and Macdonald processes » (2013), http://math.mit.edu/~icorwin/HIM_Lecture1.pdf
- A. Borodin & I. Corwin, « Macdonald processes », arXiv:1111.4408 [math.PR]

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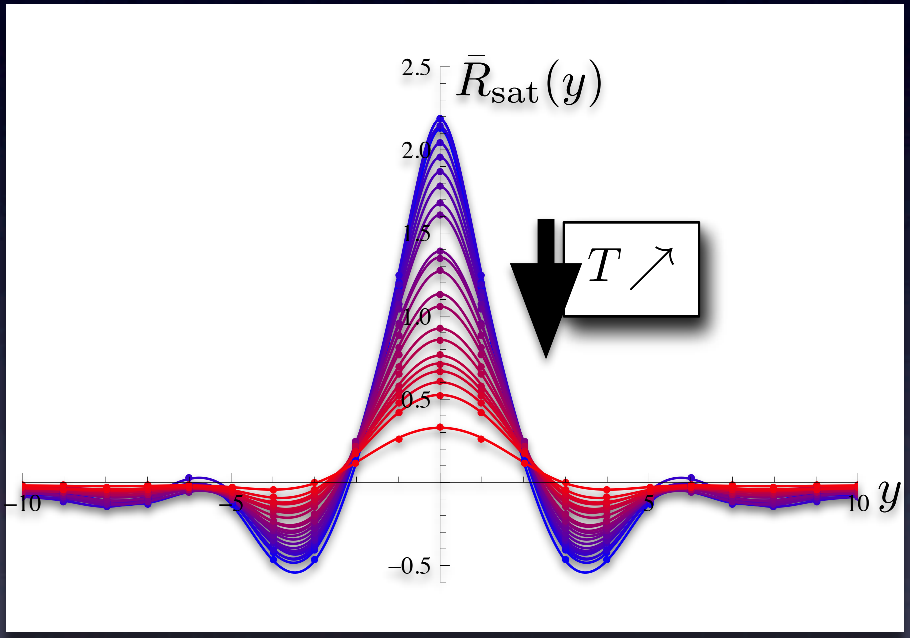
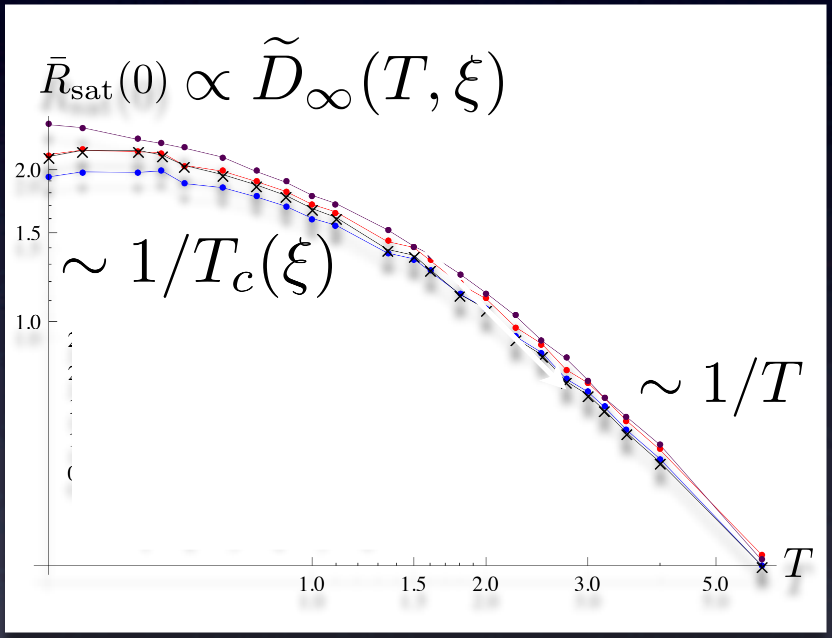
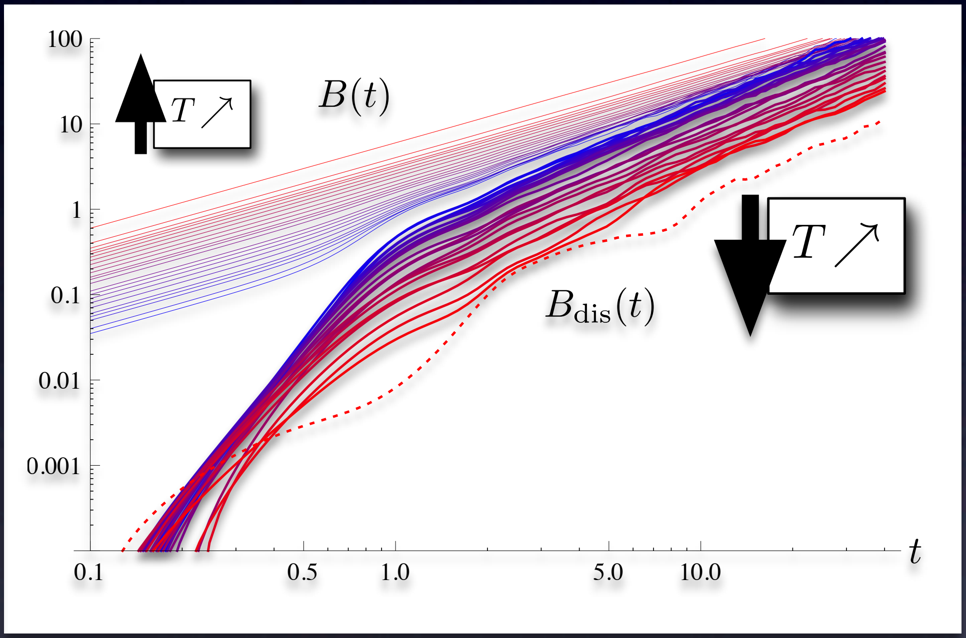


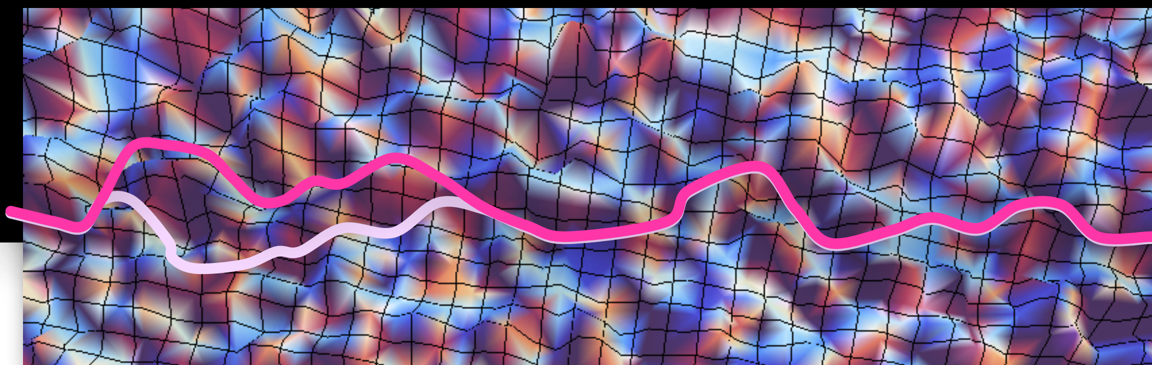


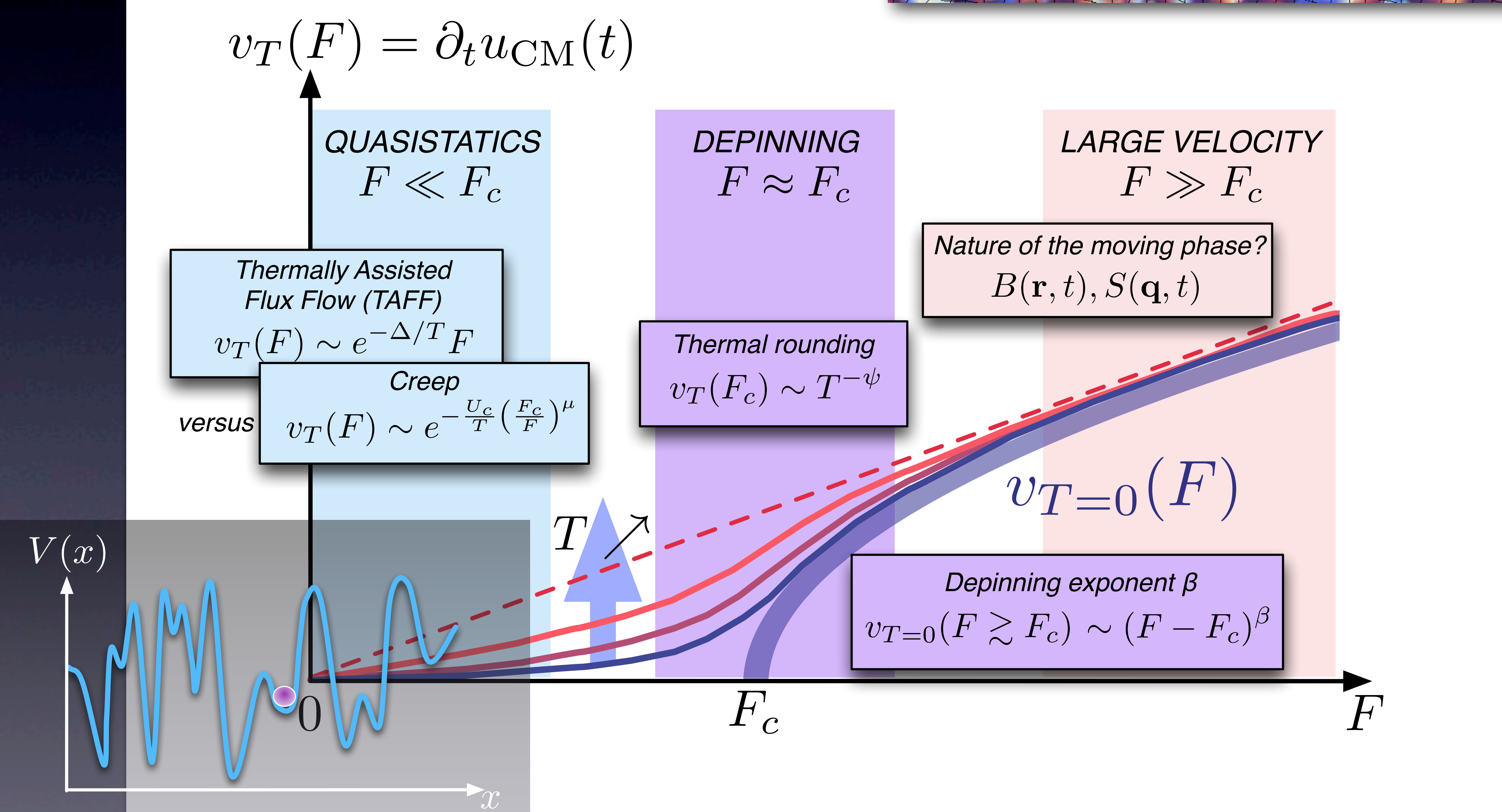


- Consequences of the low-temperature regime for the dynamics of 1D interfaces
- Connexion with the Functional-Renormalization-Group treatment of DES

DYNAMICS: How does an interface respond when pulled at it?

- Velocity-force characteristics
- Steady-state velocity of center-of-mass





Take-home messages

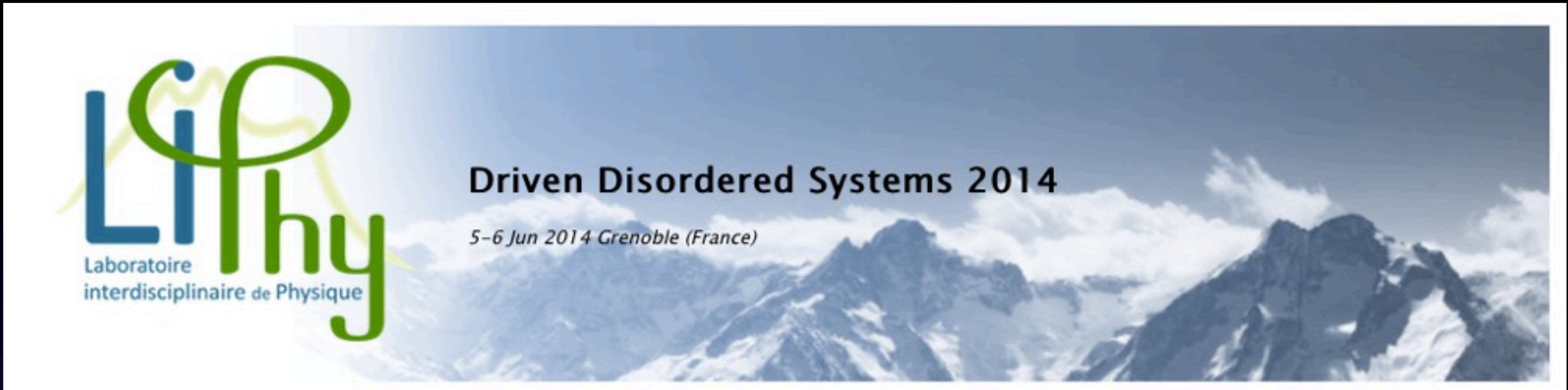
- Complex interfaces can be modelled as Disordered Elastic Systems.
- Probing disorder in STATICS/DYNAMICS depending on lengthscale
- 1D Interface width induces low-temperature statics below $T_c(\xi) = (\xi c D)^{1/3}$

Open questions

- Full dependance on the DES parameters (e.g. temperature): microscopic level?
- Internal structure?

V. Lecomte, S. E. Barnes, J.-P. Eckmann & T. Giamarchi, *Phys. Rev. B* **80**, 054413 (2009).

- Bubbles & overhangs?
- Analogy with other disordered systems (ex. amorphous materials)?



GDR PHENIX

⇒ *GDR meeting in June in Grenoble*

http://dds2014.sciencesconf.org

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