ROUGH INTERFACES
AND
ELASTIC LINES IN DISORDERED MEDIA

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EMMCSS
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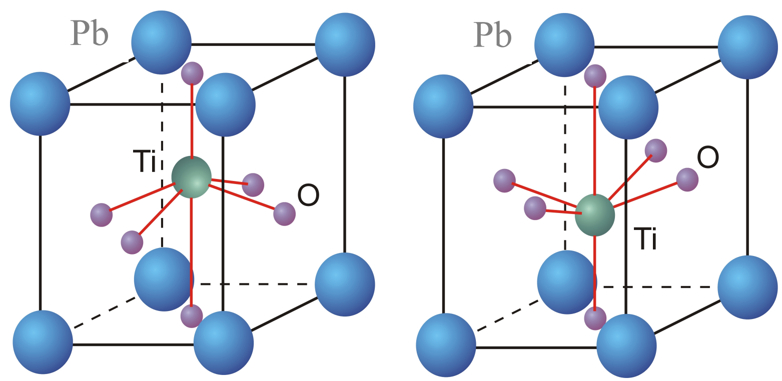
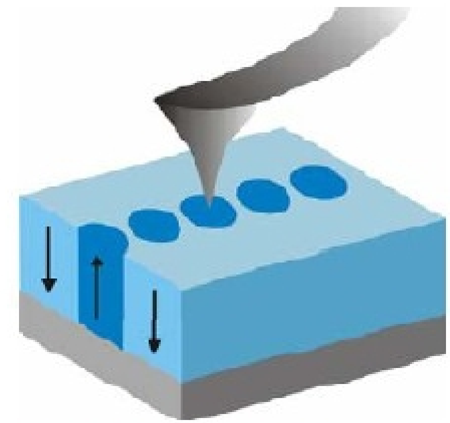
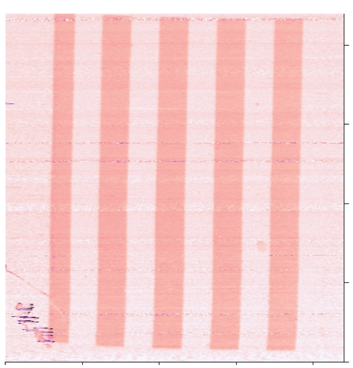
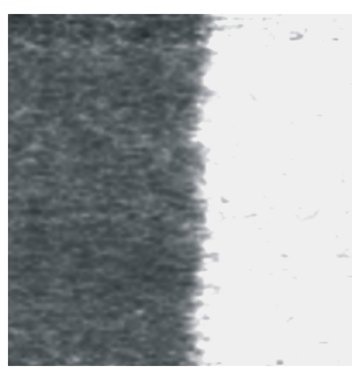
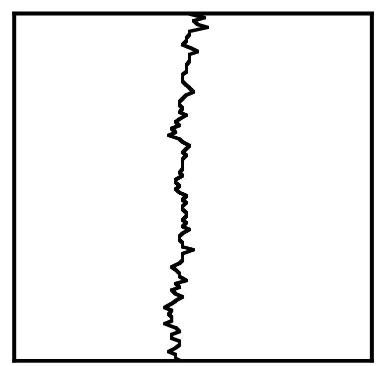
Motivation

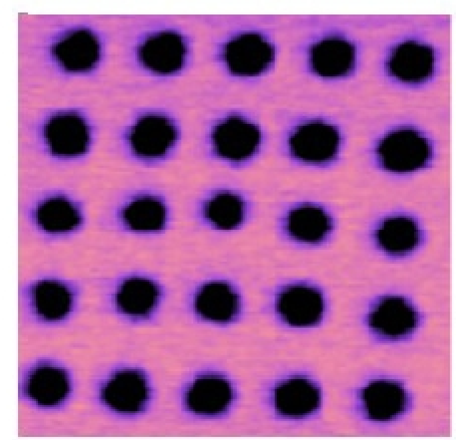




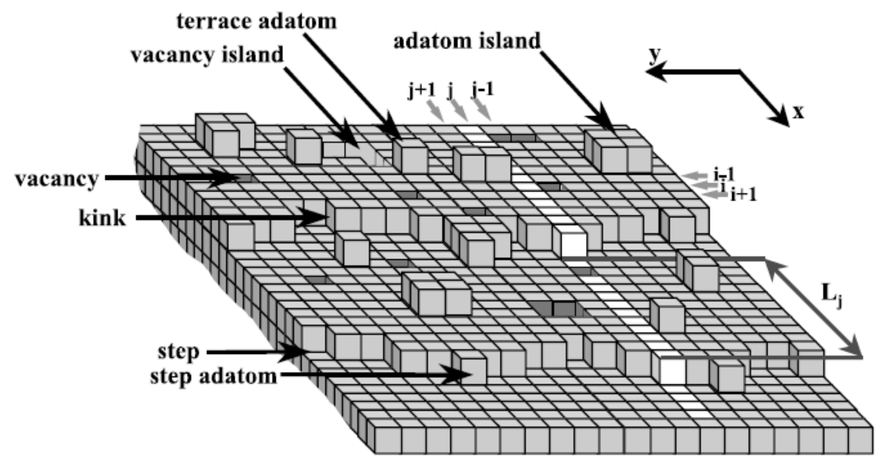




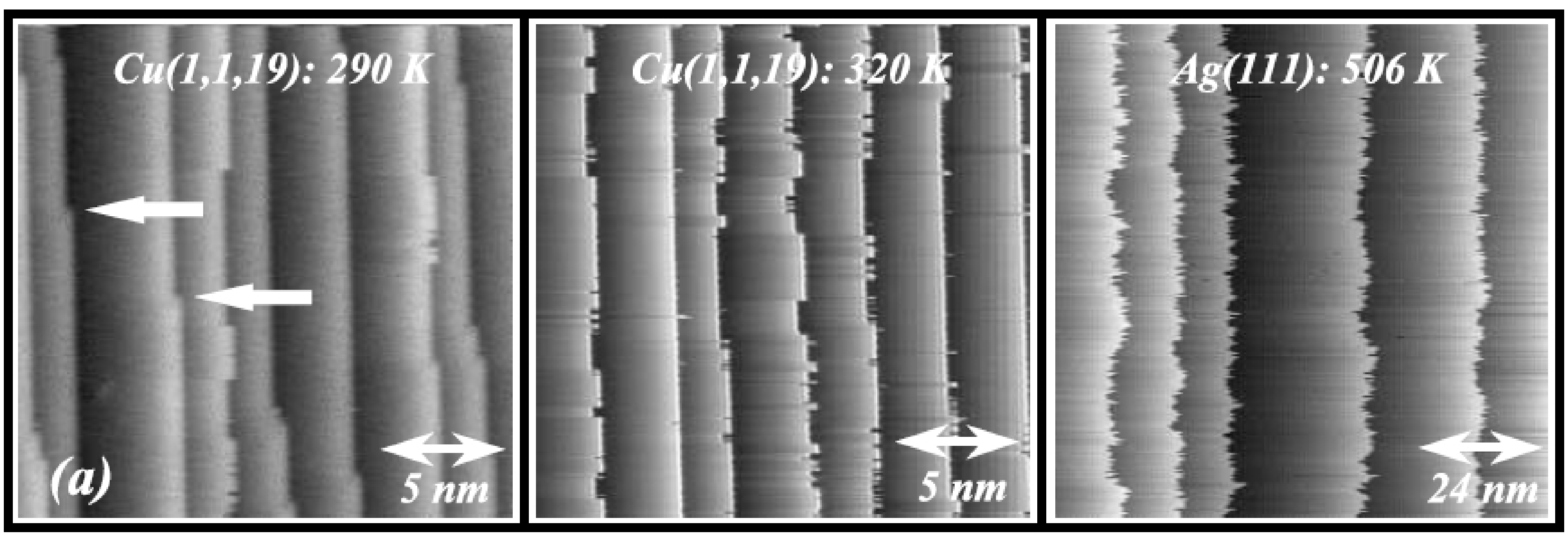




Motivation

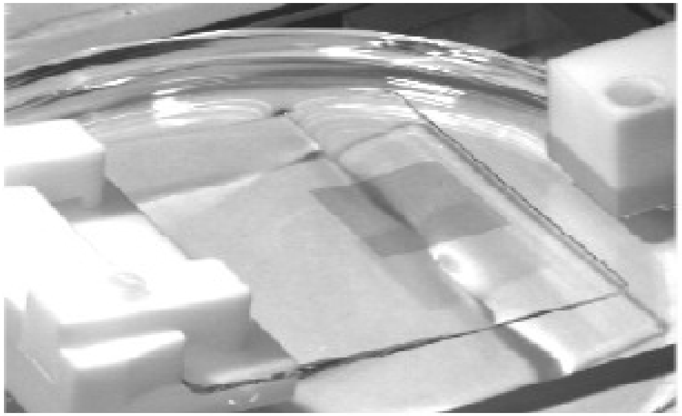


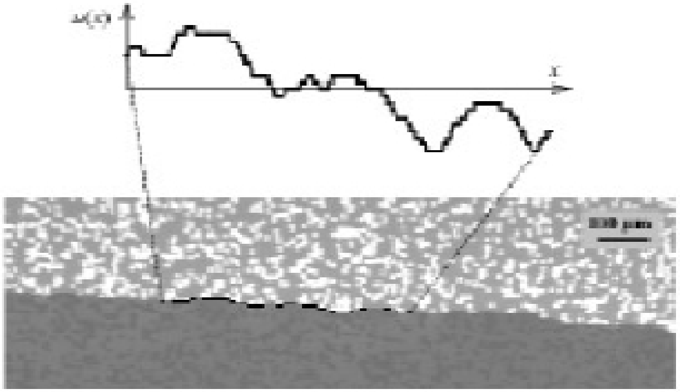
terrace adatom
vacancy island
adatom island
y
x
j+1 j j-1
i-1
i
i+1
vacancy
kink
L_j
step
step adatom



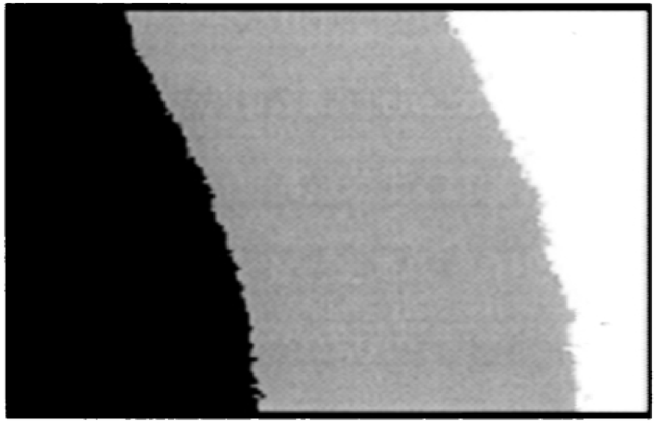
Cu(1,1,19): 290 K
5 nm
(a)
Cu(1,1,19): 320 K
5 nm
Ag(111): 506 K
24 nm

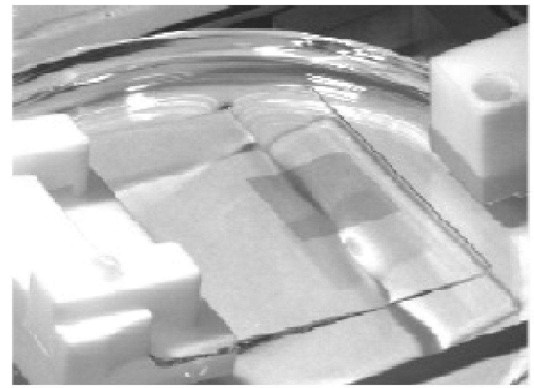
Motivation

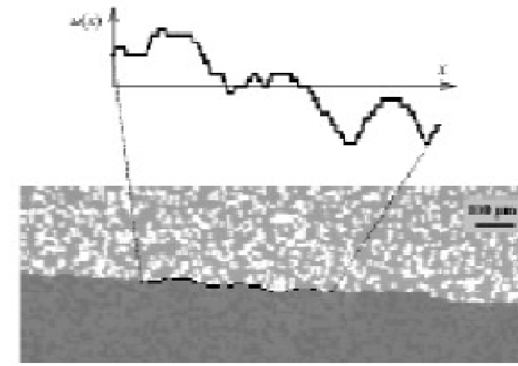




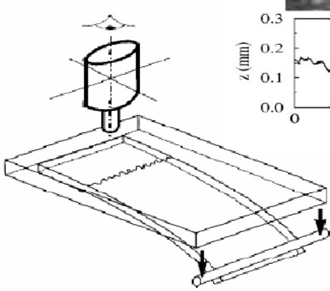
Motivation

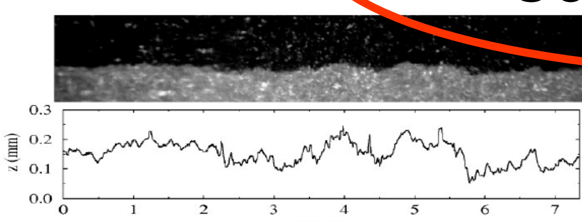


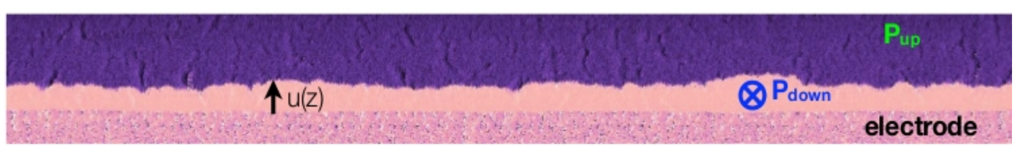




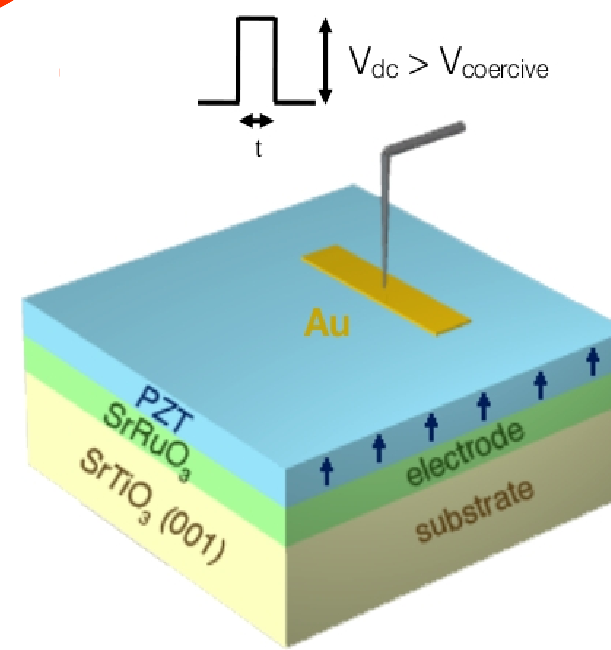
Universality
Common framework







with different writing times



Motivation

- Domain walls in ferroic materials (ferromagnets, ferroelectrics, ferroelastics...)
- Deposition of particles
- Burning fronts
- Fractures
- Cellular fronts
- Vortex matter in high temperature superconductors
- Wetting
- ...

A common framework can be used to describe universal features:
DISORDERED ELASTIC SYSTEMS

Rough interfaces
and
Elastic lines in disordered systems

- Geometrical properties
  - Fluctuations: Roughness
  - Family-Vicsek scaling

- Continuum equations
  - Edwards-Wilkinson
  - Kardar-Parisi-Zhang
  - Universality

- More on geometrical properties
  - Correlation functions
  - Anomalous scaling

- Quenched disorder
  - Quenched disorder
  - Directed polymer
  - Thermal effects
  - Depinning transition
  - Avalanches

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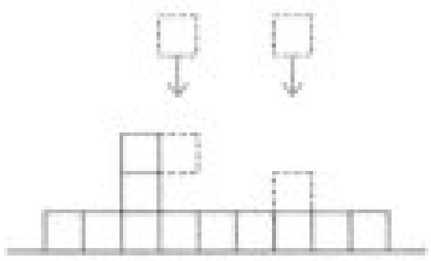
Fluctuations: roughness

Growing interfaces: discrete models



Ballistic deposition

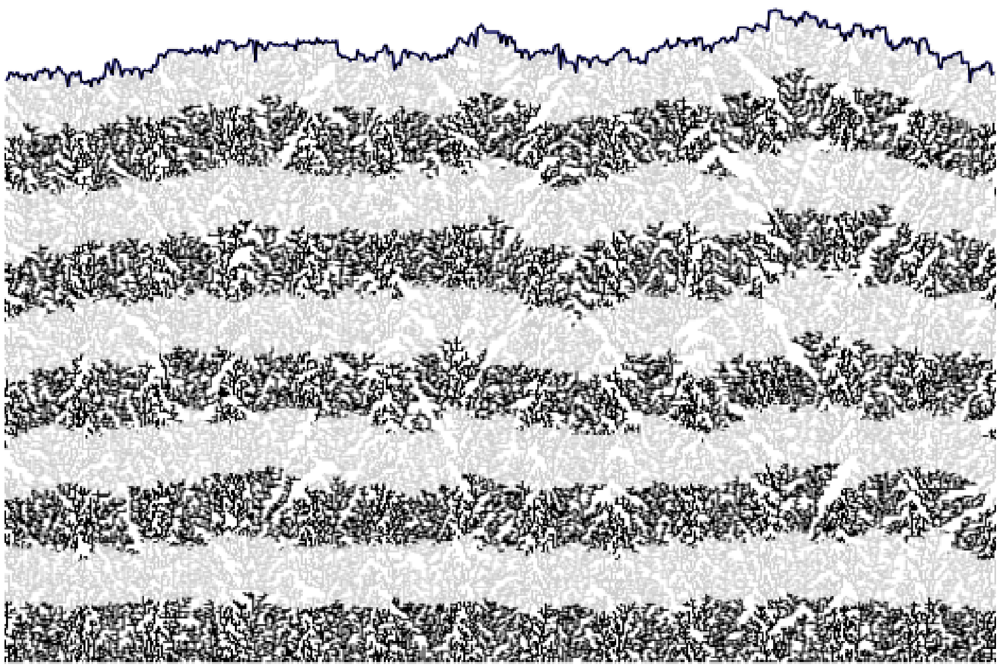
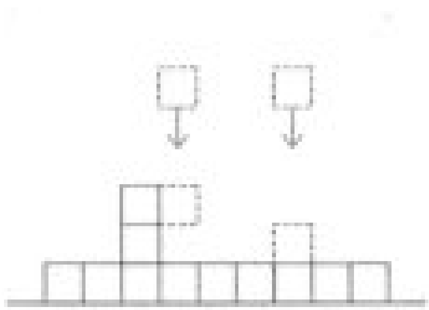


Fig. 2 – A BD cluster obtained by depositing 100,000 particles on a substrate of size $L = 512$. A time step is defined by a deposition of a single particle. The different shadings correspond to different time intervals each corresponding to the deposition of 10,000 particles.

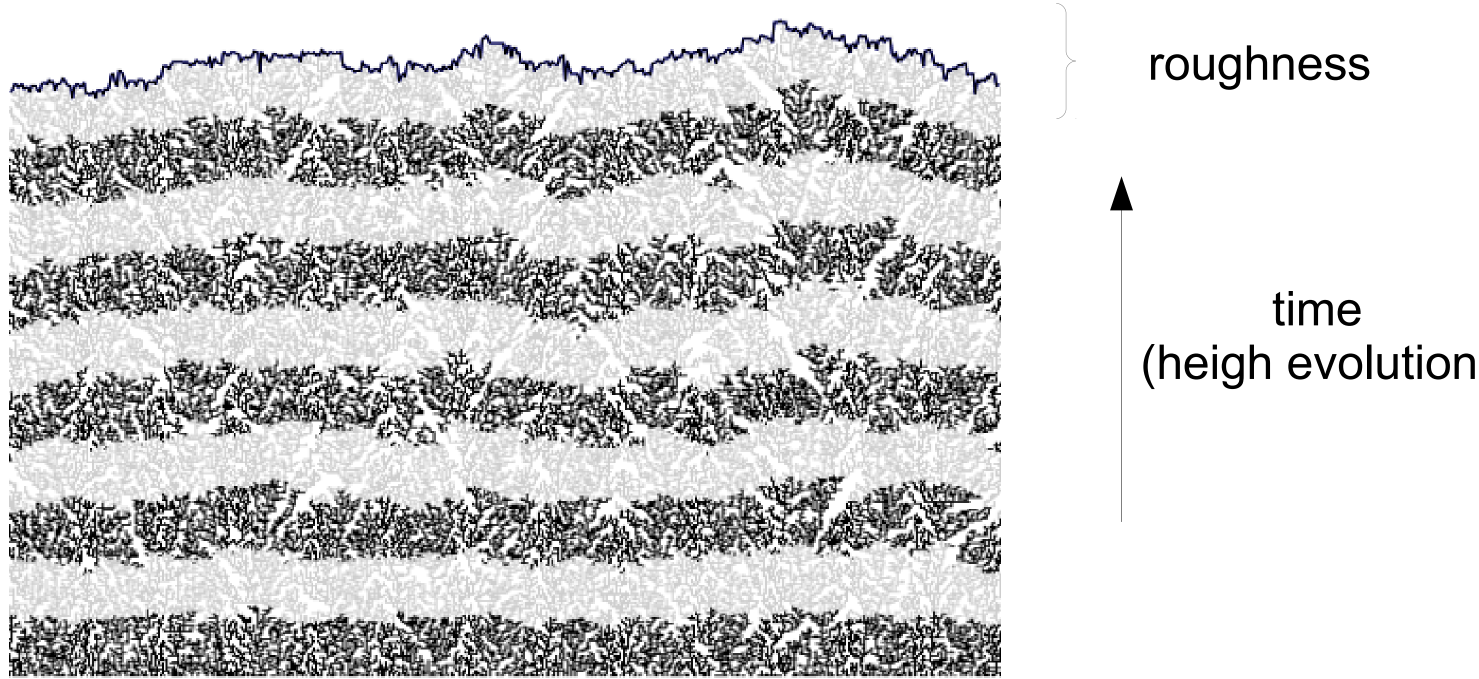
Katzav, Edwards, Schwartz, EPL **75**, 29, 2006

Fluctuations: roughness

Growing interfaces: discrete models

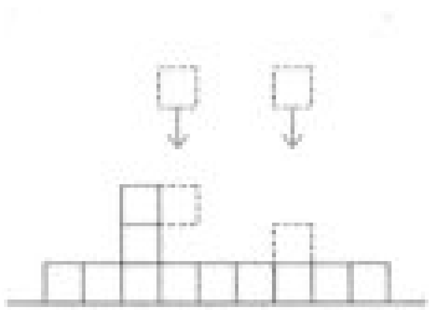


Ballistic deposition

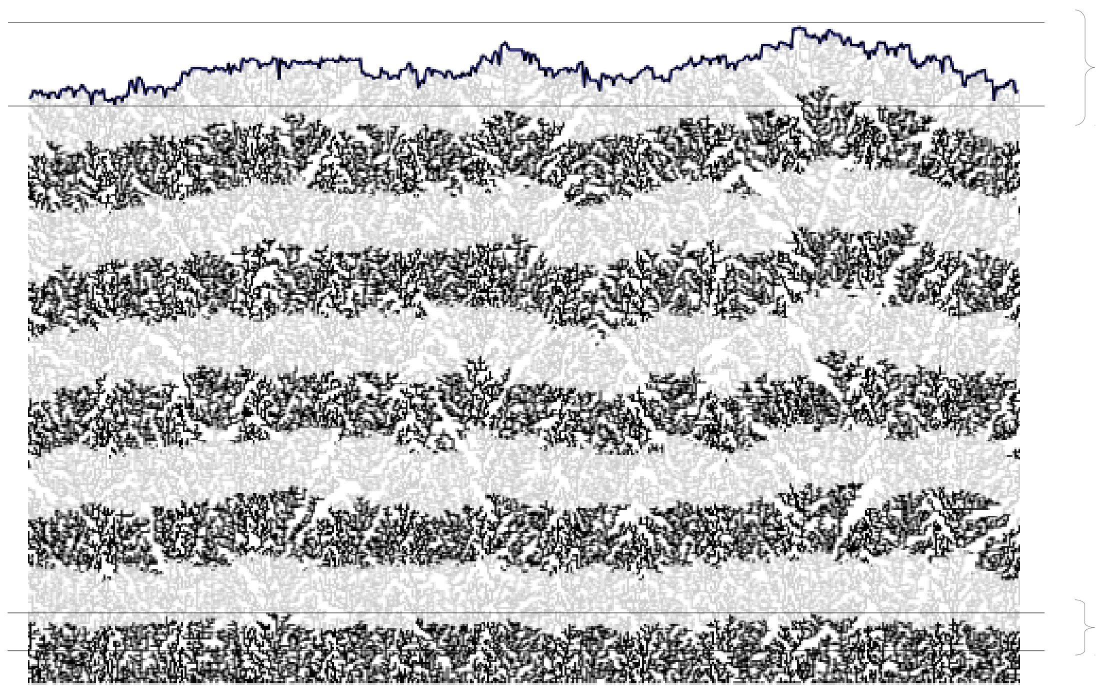


Fluctuations: roughness

Growing interfaces: discrete models



Ballistic deposition

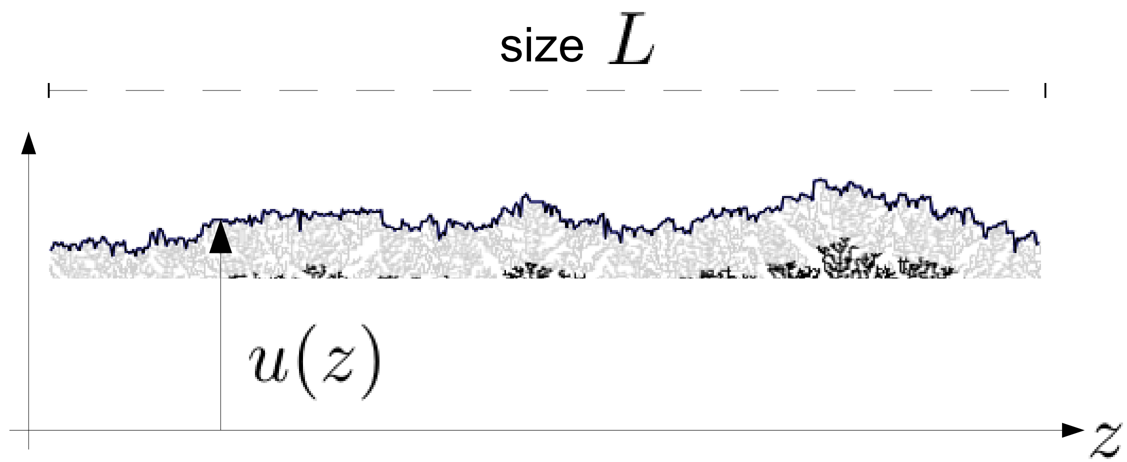


roughness

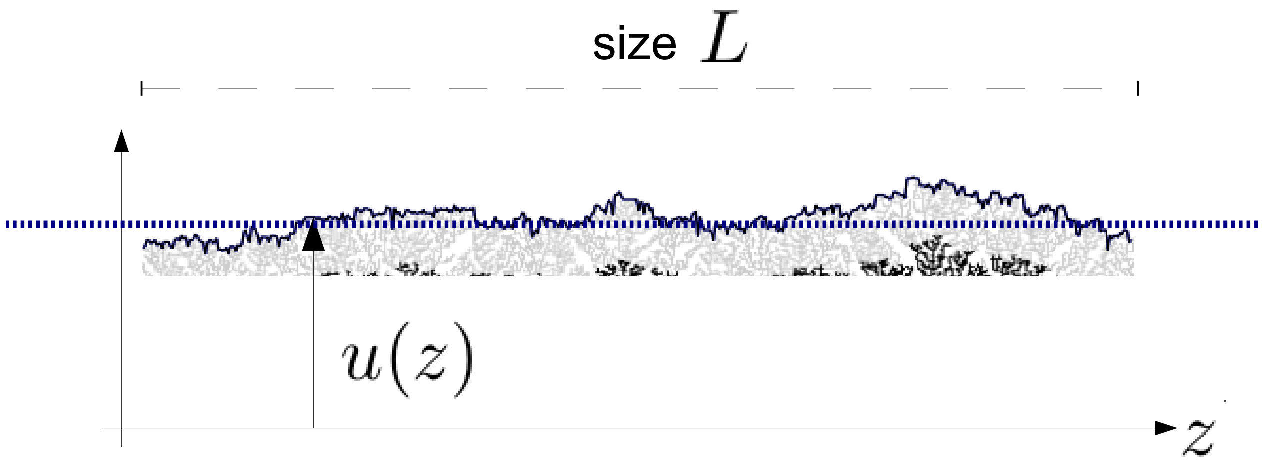
the roughness is evolving with time

roughness

Fluctuations: roughness

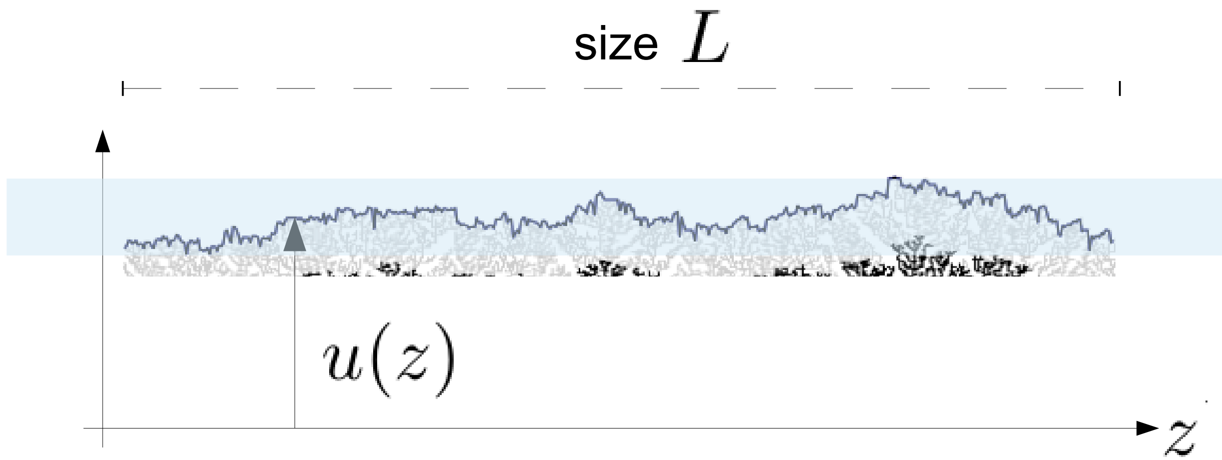


Fluctuations: roughness



mean value $$\langle u(z) \rangle = \sum_{z=0}^{L-1} u(z)$$

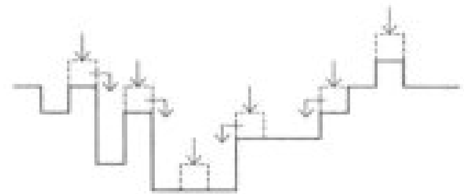
Fluctuations: roughness



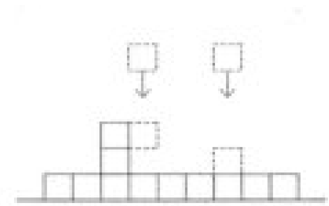
mean value $$\langle u(z) \rangle = \sum_{z=0}^{L-1} u(z)$$

roughness $$W^2 = \sum_{z=0}^{L-1} \left[u(z) - \langle u(z) \rangle \right]^2$$

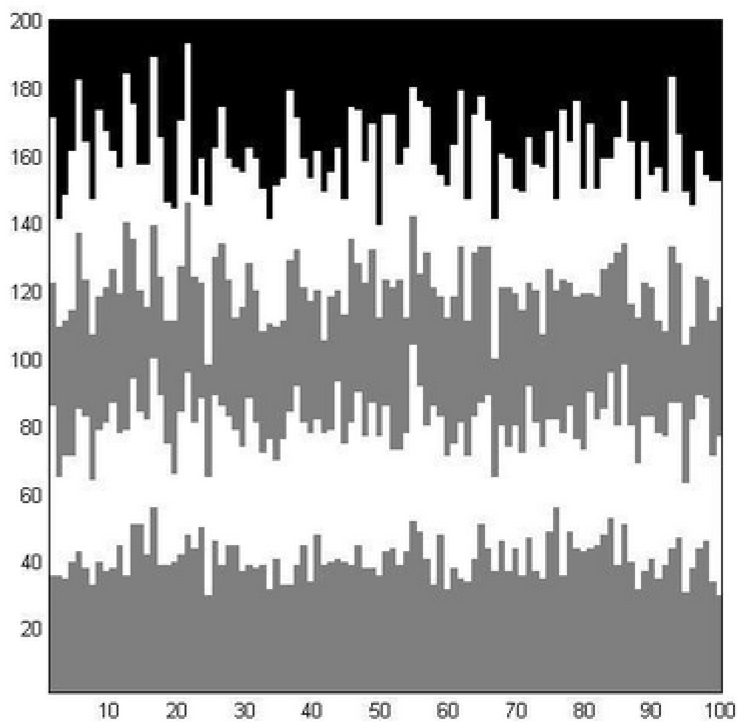
Fluctuations: roughness

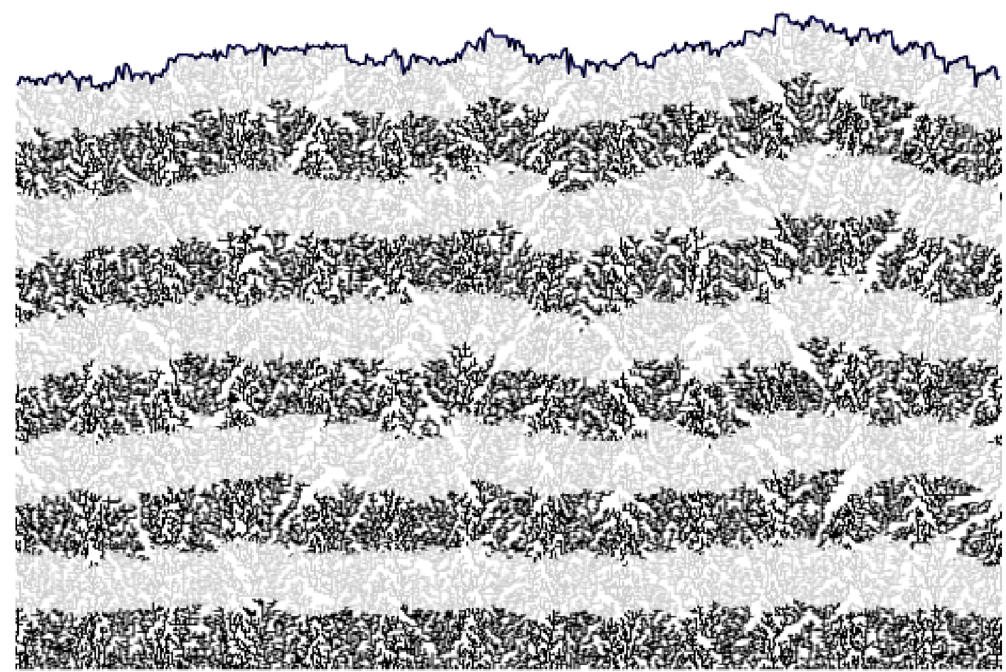


random deposition
(with surface relaxation)



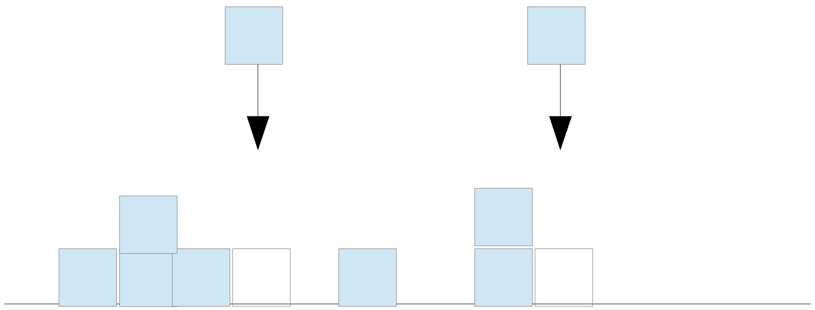
ballistic deposition





Fluctuations: roughness

random deposition



- *L* sites
- equal probability *p*=1/*L* of attaching to any site
- after *N* attachment events, time *t*=*N*/*L*

$$P(u,N) = \binom{N}{u} p^u (1-p)^{N-u}$$

$$\langle u \rangle = \sum_{u=1}^{N} u P(u,N) = Np = \frac{N}{L} = \boxed{t}$$

the interface grows at a constant speed

$$\langle u^2 \rangle = \sum_{u=1}^{N} u^2 P(u,N) = N^2p^2 + Np(1-p)$$

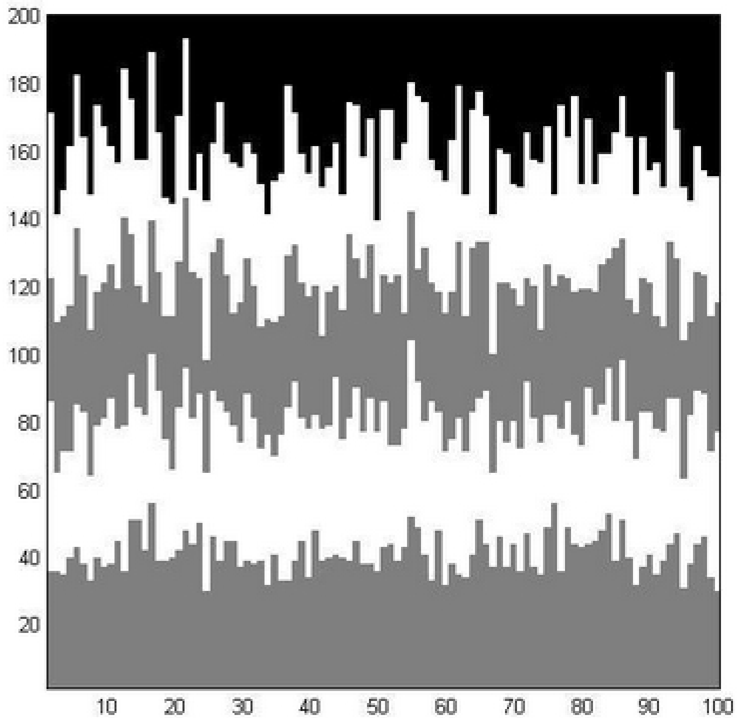
$$W^2 = \langle u^2 \rangle - \langle u \rangle^2 = Np(1-p) = \boxed{t\left(1-\frac{1}{L}\right)}$$

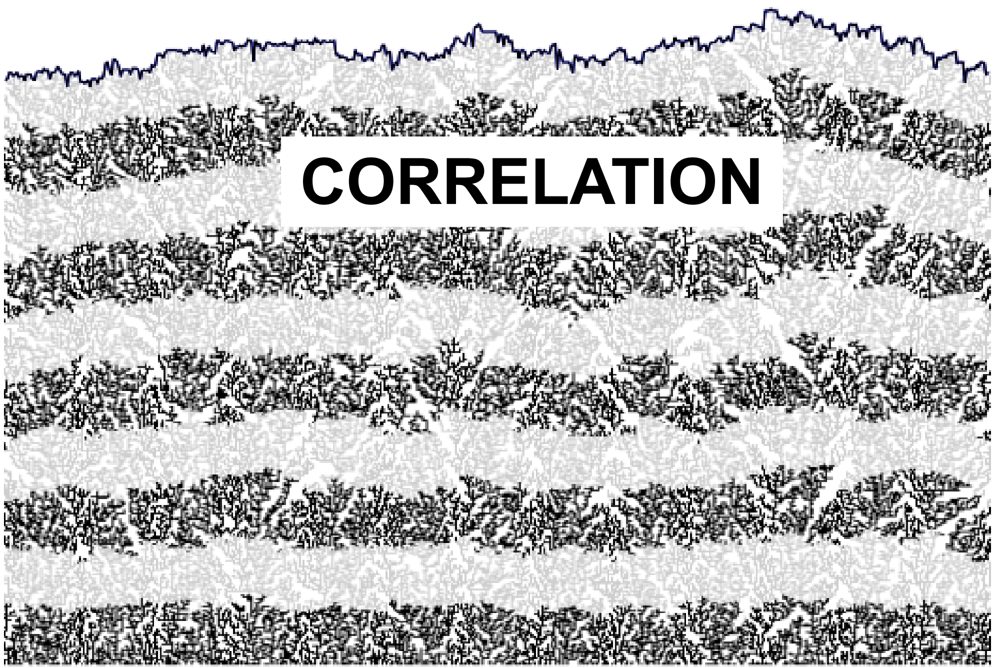
the roughness grows indefinitely

Fluctuations: roughness

$$W^2 = t\left(1 - \frac{1}{L}\right)$$

?





Family Vicsek scaling

In the general case

$$W(t,L) \sim t^{\beta} \qquad t \ll t_x$$

with β the **growing exponent**

$$W(t,L) \sim L^{\alpha} \qquad t \gg t_x$$

with α the **roughness exponent**

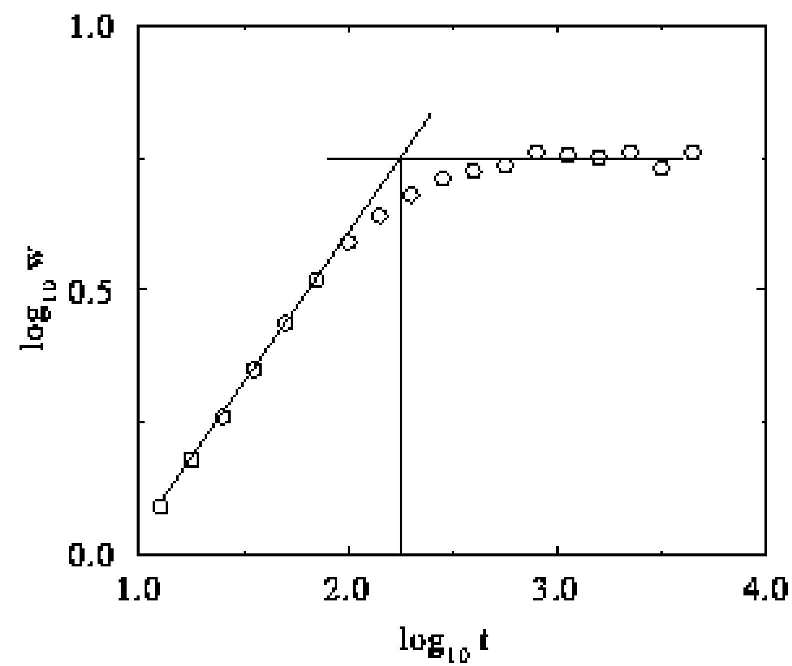
the crossover time $t_x \sim L^z$

with z the **dynamical exponent**

$$t_x^{\beta} \sim L^{z\beta} \sim L^{\alpha}$$

we have the scaling relation $z = \dfrac{\alpha}{\beta}$

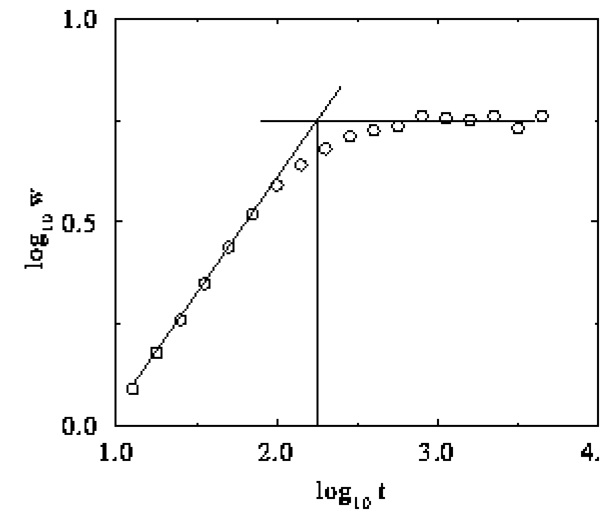
ballistic deposition

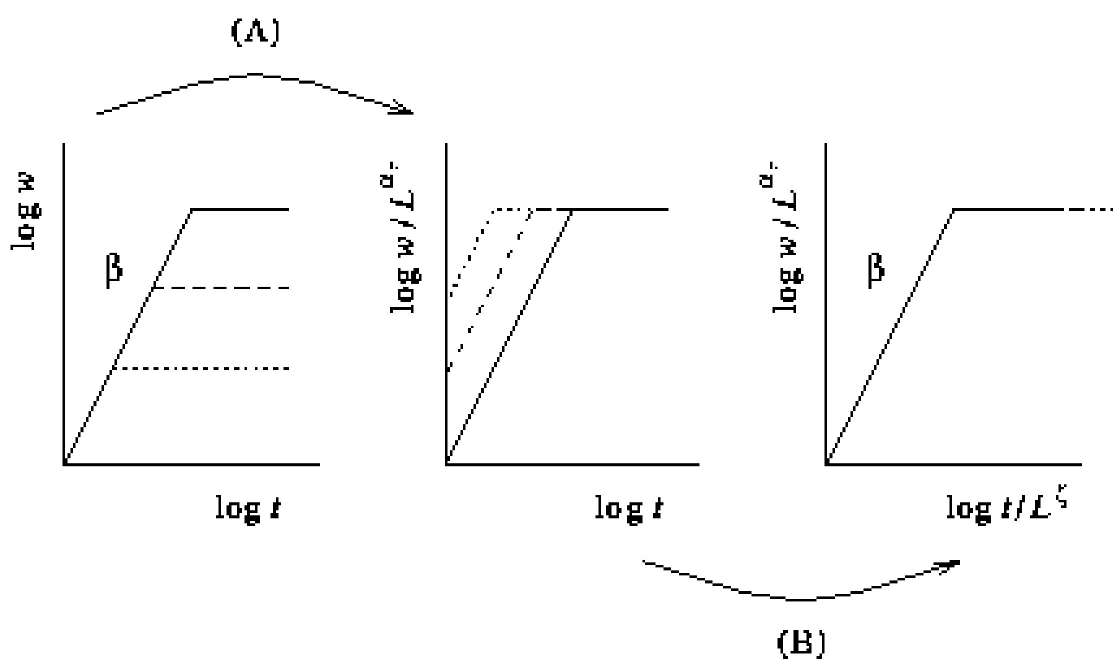


Family Vicsek scaling

(A) Plot $$\frac{\log W}{L^{\alpha}} \quad \text{vs.} \quad t$$

(B) Plot $$\frac{\log W}{L^{\alpha}} \quad \text{vs.} \quad \frac{t}{L^{z}}$$





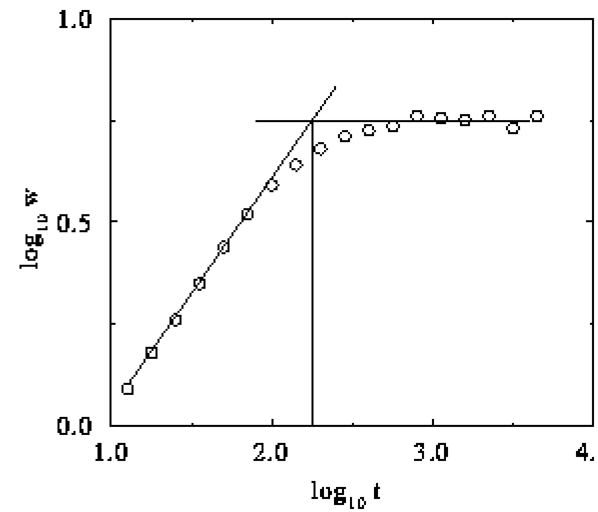
universal function

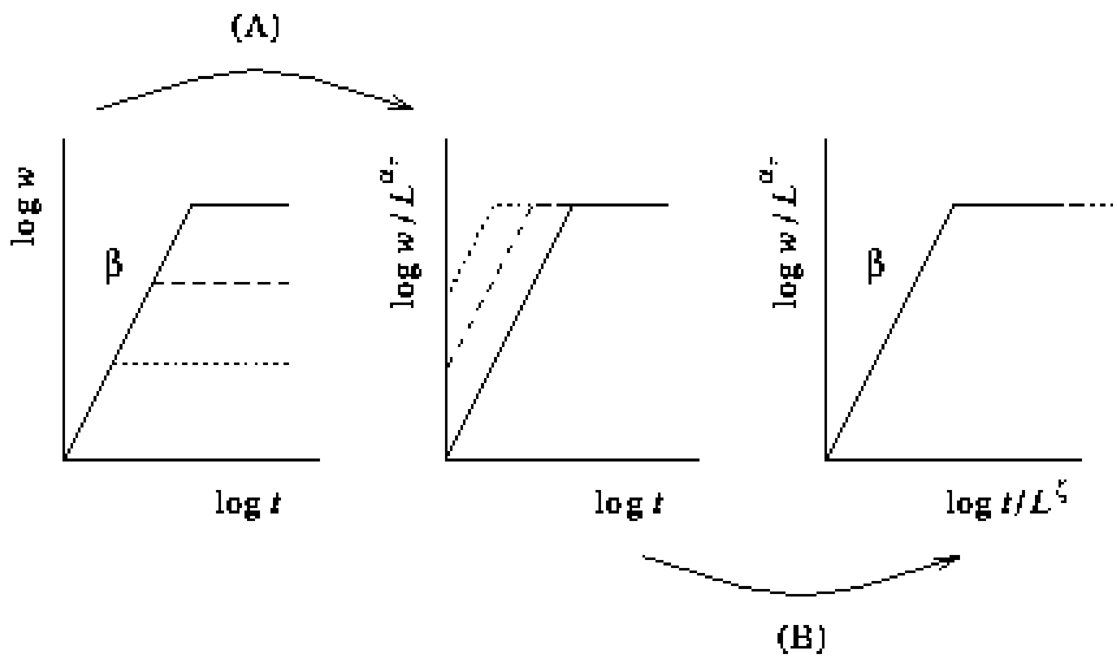
Family Vicsek scaling

$$W(t,L) = L^{\alpha} f\left(\frac{t}{L^{z}}\right)$$

$$f(x) \sim \begin{cases} x^{\beta} & \text{for } x \ll 1 \\ 1 & \text{for } x \gg 1 \end{cases}$$

scaling function



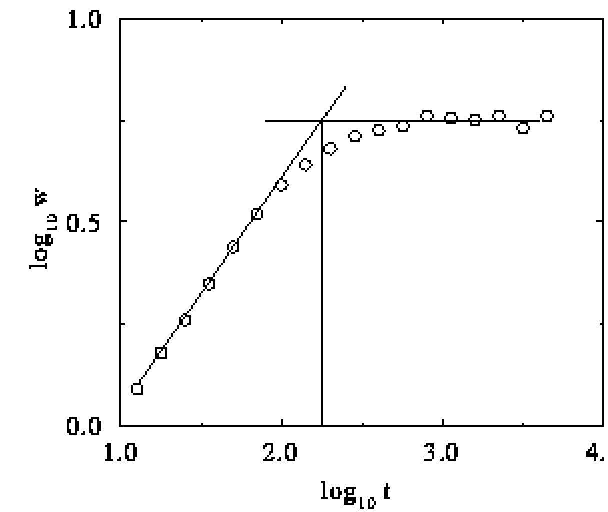


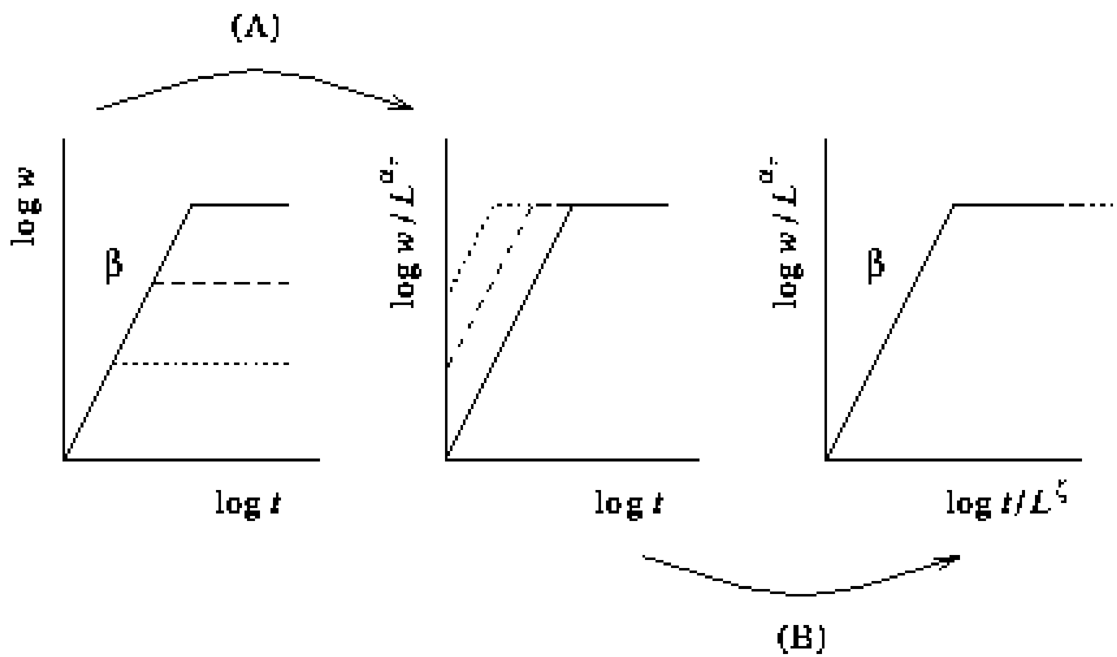
Family Vicsek scaling

$$W(t,L) = L^{\alpha} f\left(\frac{t}{L^{z}}\right)$$

$$f(x) \sim \begin{cases} x^{\beta} & \text{for } x \ll 1 \\ 1 & \text{for } x \gg 1 \end{cases}$$

transverse correlation length $\xi(t) \sim t^{1/z}$





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Edwards-Wilkinson equation

random deposition $$\frac{\partial u(z,t)}{\partial t} = \Phi(z,t)$$

$\Phi(z,t)$: position dependent flux

$$\Phi(z,t) = F + \eta(z,t)$$

F : net flux

$$\begin{aligned} \langle \eta(z,t) \rangle &= 0 \\ \langle \eta(z,t)\eta(z',t') \rangle &= D\delta(z-z')\delta(t-t') \end{aligned}$$

: noise

$$\langle u(z,t) \rangle = \left\langle \int \Phi(z,t)\ dt \right\rangle = Ft + \int \langle \eta(z,t) \rangle dt$$

$$\langle u(t) \rangle = Ft$$

$$\begin{aligned} \langle u(z,t)u(z,t') \rangle &= \left\langle \int \int \Phi(z,t)\Phi(z,t')\ dt\ dt' \right\rangle \\ &= (Ft)^2 + \int \int \langle \eta(z,t)\eta(z,t') \rangle dt\ dt' \\ &= (Ft)^2 + Dt \end{aligned}$$

$$\langle u(t)^2 \rangle = (Ft)^2 + Dt$$

Edwards-Wilkinson equation

random deposition $$\frac{\partial u(z,t)}{\partial t} = \Phi(z,t)$$

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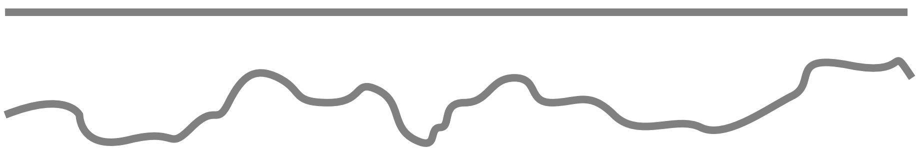
$$\langle u(t) \rangle = Ft \qquad \langle u(t)^2 \rangle = (Ft)^2 + Dt$$

$$\boxed{W^2 = Dt}$$

$$\beta = 1/2 \quad \alpha = 0$$

position and size independent – **no correlations**

Edwards-Wilkinson equation



We consider now that there is a cost in roughening the interface

Hamiltonian approach

$$\mathcal{H} = \int_{\mathcal{L}} c ds = c \int_{\mathcal{L}} \sqrt{dz^2 + du^2} = c \int_{\mathcal{L}} \sqrt{1 + (du/dz)^2} dz$$

$$\begin{aligned} \mathcal{H} &= c \int_L \sqrt{1 + \left(\frac{\partial u}{\partial z}\right)^2} dz \\ &= c \int_L \left[1 + \frac{1}{2}\left(\frac{\partial u}{\partial z}\right)^2 - \frac{1}{8}\left(\frac{\partial u}{\partial z}\right)^4 + \frac{1}{16}\left(\frac{\partial u}{\partial z}\right)^6 + \dots\right] dz \end{aligned}$$

$$\mathcal{H}_{\mathrm{el}} = \frac{c}{2} \int_L \left(\frac{\partial u}{\partial z}\right)^2 dz$$

elastic enery contribution

Edwards-Wilkinson equation

$$\gamma \frac{\partial u(z,t)}{\partial t} = -\frac{\delta \mathcal{H}[u(z,t)]}{\delta u(z,t)} + \eta(z,t)$$

Langevin dynamics
- non-conserved dynaimc equation
- Model A
- overdamped equation of motion

$$\begin{aligned} \langle \eta(z,t) \rangle &= 0 \\ \langle \eta(z,t)\eta(z',t') \rangle &= 2\gamma T \delta(z-z')\delta(t-t') \end{aligned}$$

white noise

$\frac{\delta \mathcal{H}}{\delta u}$ functional derivative

Property: **if** $\mathcal{H} = \int f\left(u, \frac{\partial u}{\partial z}\right) dz$

then $\frac{\delta \mathcal{H}}{\delta u} = \frac{\partial f}{\partial u} - \frac{\partial}{\partial z}\frac{\partial f}{\partial(\partial_z u)}$

therefore: $\mathcal{H}_{\mathrm{el}} = \frac{c}{2}\int_L \left(\frac{\partial u}{\partial z}\right)^2 dz \longrightarrow \frac{\delta \mathcal{H}_{\mathrm{el}}}{\delta u} = -c\frac{\partial^2 u}{\partial z^2}$

Edwards-Wilkinson equation

$$\gamma \frac{\partial u(z,t)}{\partial t} = -\frac{\delta \mathcal{H}[u(z,t)]}{\delta u(z,t)} + \eta(z,t)$$

Langevin dynamics
- non-conserved dynaimc equation
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$$\begin{aligned} \langle \eta(z,t) \rangle &= 0 \\ \langle \eta(z,t)\eta(z',t') \rangle &= 2\gamma T \delta(z-z')\delta(t-t') \end{aligned}$$

white noise

$$\frac{\partial u(z,t)}{\partial t} = \nu \frac{\partial^2 u(z,t)}{\partial z^2} + \eta(z,t)$$

Edwards-Wilkinson equation

$$\begin{aligned} \langle \eta(z,t) \rangle &= 0 \\ \langle \eta(z,t)\eta(z',t') \rangle &= \frac{2T}{\gamma} \delta(z-z')\delta(t-t') \end{aligned}$$

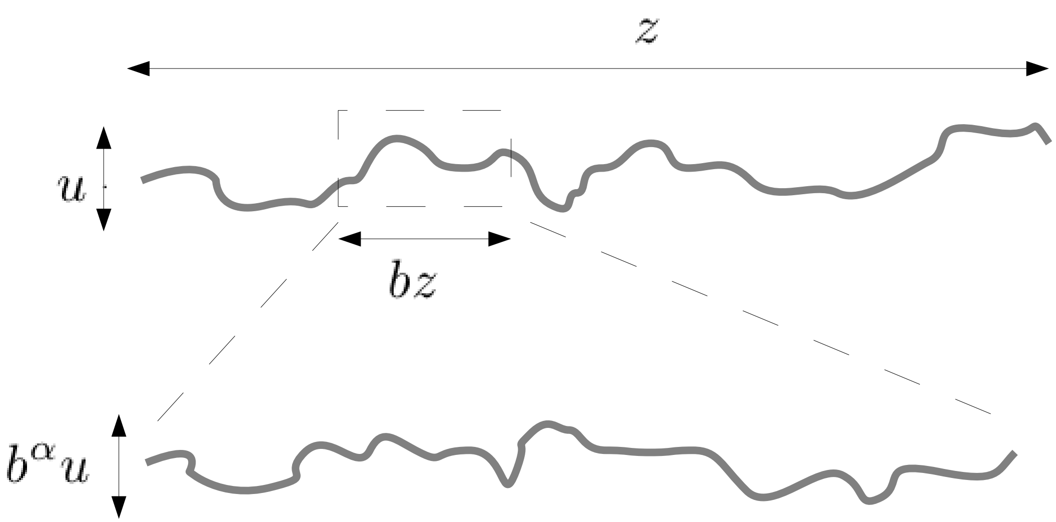
Edwards and Wilkinson, Proc. R. Soc.. **381**, 17, 1982

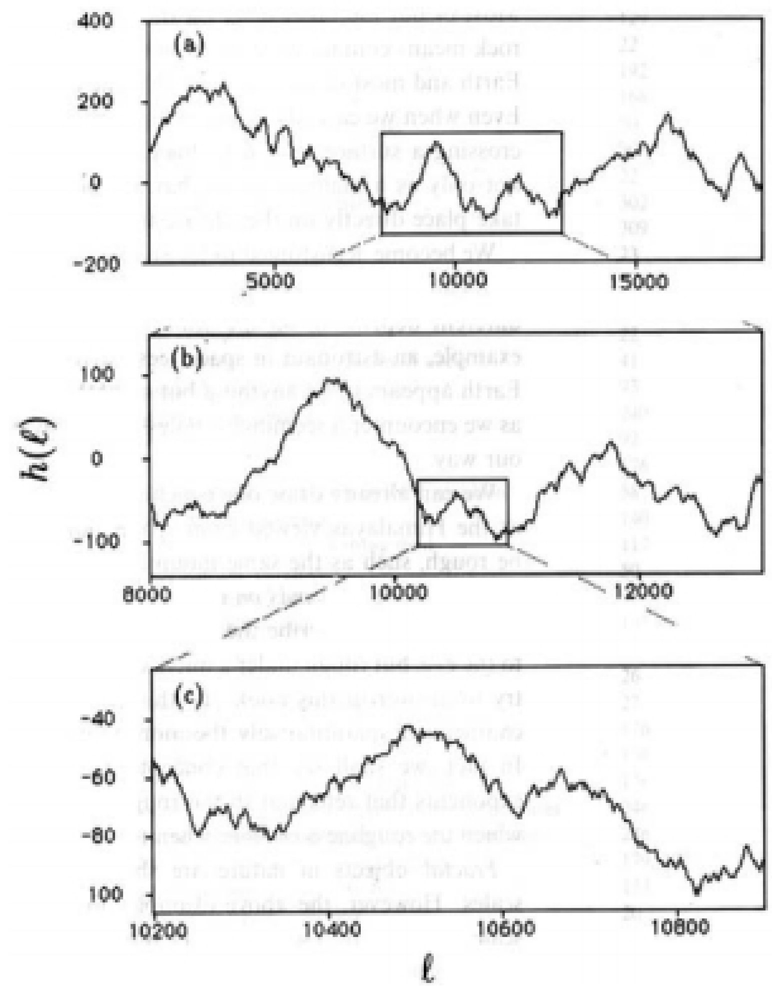
Edwards-Wilkinson equation

self-affinity: the interface is **statistically** invariant under an anisotropic transformation

$$z \to z' = bz$$

$$u \to u' = b^{\alpha} u$$





dynamically, statistically invariance is recovered using $t \to t' = b^{z} t$

Edwards-Wilkinson equation

$$z \to z' = bz \qquad u \to u' = b^\alpha u \qquad t \to t' = b^z t$$

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \nu \frac{\partial^2 u}{\partial z^2} + \eta(z,t) \\
\frac{\partial (b^\alpha u)}{\partial (b^z t)} &= \nu \frac{\partial^2 (b^\alpha u)}{\partial (bz)^2} + \eta(bz, b^z t) \\
b^{\alpha - z} \frac{\partial u}{\partial t} &= \nu b^{\alpha-2} \frac{\partial^2 u}{\partial z^2} + b^{-1/2 - z/2} \eta(z,t) \\
\frac{\partial u}{\partial t} &= \nu b^{z-2} \frac{\partial^2 u}{\partial z^2} + b^{-1/2 + z/2 + \alpha} \eta(z,t)
\end{aligned}$$

$$z - 2 = 0$$

$$-1/2 + z/2 + \alpha = 0$$

$$z = 2 \quad \alpha = 1/2 \quad \beta = 1/4$$

EW exponentes

Edwards-Wilkinson equation

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} + \eta(z,t)$$

linear, partial derivatives equation
it can be solved!

Fourier representation

$$\delta u(z,t) = u(z,t) - \overline{u}(t) = \sum_{n=-\infty}^{\infty} c_n(t) e^{i q_n t}$$

with $q_n = \frac{2\pi n}{L}$

where $\overline{u}(t) = L^{-1} \int_0^L u(z,t)\, dz$ and $c_n(t) = L^{-1} \int_0^L \delta u(z,t) e^{-i q_n t} dz$

$$\frac{\partial c_n(t)}{\partial t} = -\nu q_n^2 c_n(t) + \eta_n(t)$$

$$\begin{aligned} \langle \eta_n(t) \rangle &= 0 \\ \langle \eta_n(t) \eta_{n'}(t') \rangle &= \frac{2T}{\gamma L} \delta_{n,-n'} \delta(t-t') \end{aligned}$$

Edwards-Wilkinson equation

$$\frac{\partial c_n(t)}{\partial t} = -\nu q_n^2 c_n(t) + \eta_n(t)$$

$$\begin{aligned} \langle \eta_n(t) \rangle &= 0 \\ \langle \eta_n(t) \eta_{n'}(t') \rangle &= \frac{2T}{\gamma L} \delta_{n,-n'} \delta(t-t') \end{aligned}$$

the solution is $$c_n(t) = c_0(0) e^{-\nu q_n^2 t} + e^{-\nu q_n^2 t} \int_0^t e^{\nu q_n^2 t'} \eta_n(t') dt'$$

The roughness can be written as $$W^2(t) = 2 \sum_{n=1}^{\infty} \langle |c_n(t)|^2 \rangle$$

with the flat initial condition
$c_n(0) = 0 \quad \forall n$

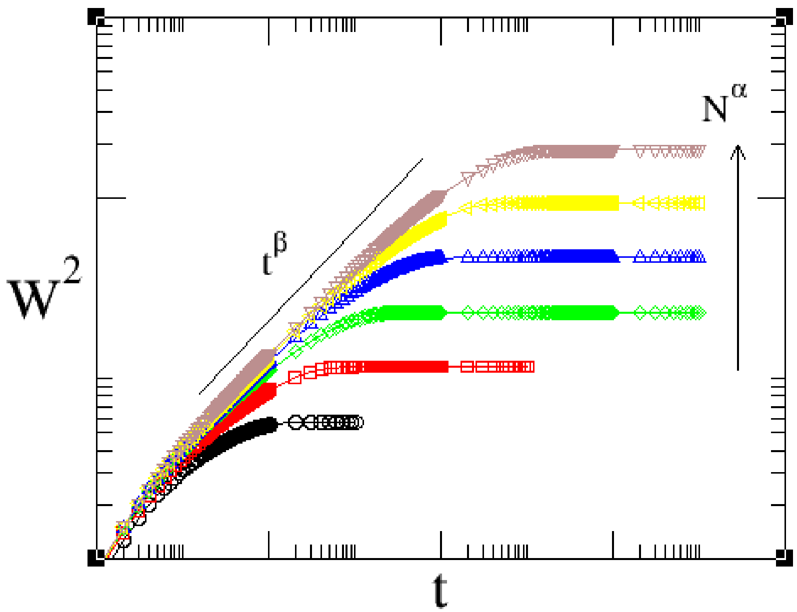
$$W^2(t) = \frac{TL}{2\pi^2 \gamma \nu} \sum_{n=1}^{\infty} \frac{1}{n^2} \left(1 - e^{n^2 t/t_x}\right)$$

$$t_x = \frac{L^2}{8\pi^2 \nu}$$

growing correlation lenght

$$\xi(t) = L\sqrt{1 - e^{-t/t_x}} \sim \begin{cases} t^{1/2} & \text{for } t \ll t_x \\ L & \text{for } t \gg t_x \end{cases}$$

Edwards-Wilkinson equation



$$W^2(t) \sim \begin{cases} \frac{T}{2\gamma}\sqrt{\frac{t}{\pi\nu}} & \text{for } t \ll t_x \\ \frac{TL}{12\gamma\nu} \sim L^{2\alpha} & \text{for } t \gg t_x \end{cases}$$

The roughness can be written as $$W^2(t) = 2\sum_{n=1}^{\infty}\langle|c_n(t)|^2\rangle$$

with the flat initial condition $c_n(0) = 0 \quad \forall n$

$$W^2(t) = \frac{TL}{2\pi^2\gamma\nu}\sum_{n=1}^{\infty}\frac{1}{n^2}\left(1 - e^{n^2 t/t_x}\right)$$

$$t_x = \frac{L^2}{8\pi^2\nu}$$

growing correlation lenght

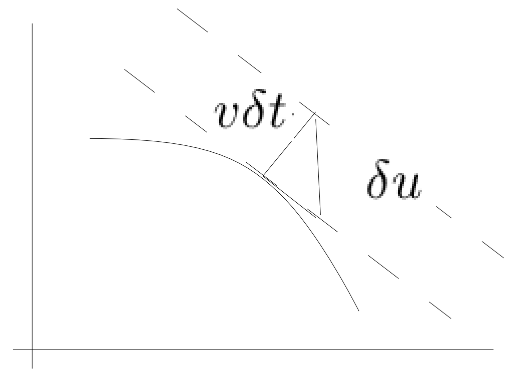
$$\xi(t) = L\sqrt{1 - e^{-t/t_x}} \sim \begin{cases} t^{1/2} & \text{for } t \ll t_x \\ L & \text{for } t \gg t_x \end{cases}$$

$z = 2 \quad \alpha = 1/2 \quad \beta = 1/4$

EW exponentes

Kardar-Parisi-Zhang equation

first non-linear correction
for lateral growing



$$\delta u \quad = \quad \sqrt{(v\delta t)^2 + (v\delta t \partial_z u)^2} = v\delta t \sqrt{1 + (\partial_z u)^2}$$

gradient
expansion

$$\frac{\partial u}{\partial t} \quad = \quad v + \frac{v}{2}\left(\frac{\partial u}{\partial z}\right)^2 + ...$$

$$\frac{\partial u(z,t)}{\partial t} = \nu \frac{\partial^2 u(z,t)}{\partial z^2} + \frac{\lambda}{2}\left(\frac{\partial u}{\partial z}\right)^2 + \eta(z,t)$$

Kardar-Parisi-Zhang equation

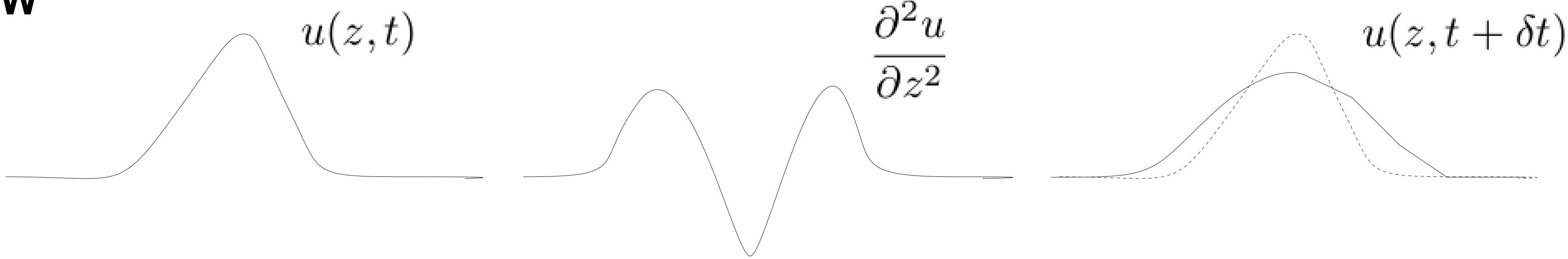
No Hamiltonian approach since the $u \to -u$ symmetry is broken

Kardar, Parisi, Zhang, PRL. **56**, 889, 1986

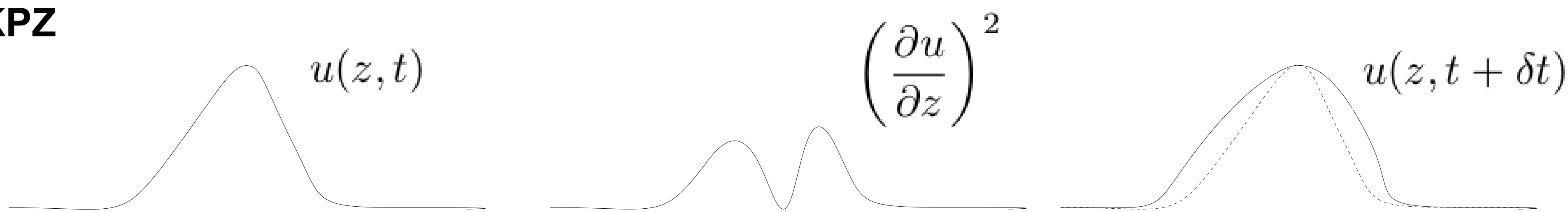
Kardar-Parisi-Zhang equation

$$\frac{\partial u(z,t)}{\partial t} = \nu \frac{\partial^2 u(z,t)}{\partial z^2} + \frac{\lambda}{2}\left(\frac{\partial u}{\partial z}\right)^2 + \eta(z,t)$$

EW



KPZ



Kardar-Parisi-Zhang equation

The KPZ equation has an intrinsic
finite velocity contribution

$$\begin{aligned}
\frac{\partial u(z,t)}{\partial t} &= \nu \frac{\partial^2 u(z,t)}{\partial z^2} + \frac{\lambda}{2}\left(\frac{\partial u}{\partial z}\right)^2 + \eta(z,t) \\
\int_L dz \frac{\partial u(z,t)}{\partial t} &= \nu \int_L dz \frac{\partial^2 u(z,t)}{\partial z^2} + \frac{\lambda}{2}\int_L dz \left(\frac{\partial u}{\partial z}\right)^2 + \int_L dz \eta(z,t) \\
\frac{\partial \overline{u}}{\partial t} &= \frac{\lambda}{2}\int_L dz \left(\frac{\partial u}{\partial z}\right)^2 > 0
\end{aligned}$$

Kardar-Parisi-Zhang equation

Langevin equation $\partial_t y = G(y) + \eta(t)$

Fokker-Planck equation $\partial_t P(y,t) = \partial_y \left[-G(y)P(y,t) + D/2\partial_y P(y,t) \right]$

$$\partial_t u = \nu \partial_z^2 u + \frac{\lambda}{2} \left(\partial_z u\right)^2 + \eta(z,t)$$

$$\partial_t P[u(z,t)] = \int dz \frac{\delta}{\delta y} \left\{ - \left[\nu \partial_z^2 u + \frac{\lambda}{2} \left(\partial_z u\right)^2 \right] P[u(z,t)] + \frac{D}{2} \frac{\delta}{\delta y} P[u(z,t)] \right\}$$

the stationary solution $\partial_t P_S[u(z,t)] = 0$ becomes

$$P_S[u(z,t)] = \exp\left[\frac{\nu}{D} \int dz \left(\partial_z u\right)^2 \right]$$

this implies that the nonlinear KPZ term is irrelevant at long times and thus that

$$\alpha_{\mathrm{KPZ}} = 1/2$$

Kardar-Parisi-Zhang equation

Galilean invariance:
the KPZ equation is invariante under

$$\begin{cases} z & \rightarrow & z - \lambda v t \\ u & \rightarrow & u + v z \\ F & \rightarrow & F - \lambda v^2/2 \end{cases}$$

$$\partial_t u = \nu \partial_z^2 u + \frac{\lambda}{2} (\partial_z u)^2 + F + \eta(z,t)$$

this implies that the nonlinear term is invariant under $z \rightarrow z' = bz \quad u \rightarrow u' = b^\alpha u$

therefore the following scaling relations holds: $z + \alpha = 2$

$$\boxed{\alpha_{\mathrm{KPZ}} = 1/2 \quad z_{\mathrm{KPZ}} = 3/2 \quad \beta_{\mathrm{KPZ}} = 1/3}$$

Universality

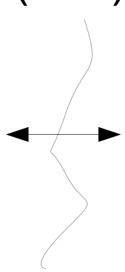
universality in terms of exponentes:
- roughness exponent α
- dynamic exponent z
- growing exponent β

interactions

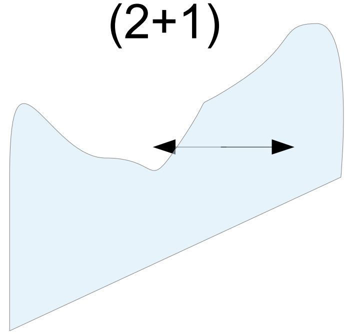
$$\partial_t u = \underbrace{\nu \partial_z^2 u}_{\text{EW}} + \underbrace{(\partial_z u)^2 \partial_z^2 u}_{\text{anharmonic}} + \underbrace{\frac{\lambda}{2} (\partial_z u)^2}_{\text{KPZ}} + \underbrace{\int \frac{u(z')}{|z - z'|} dz'}_{\text{long-range}} + \underbrace{\eta(z,t)}_{\text{noise}} + \underbrace{\xi(u,z)}_{\text{quenched noise}} + \dots$$

dimensionality

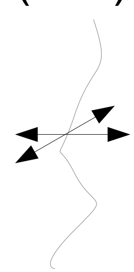
(1+1)



(2+1)

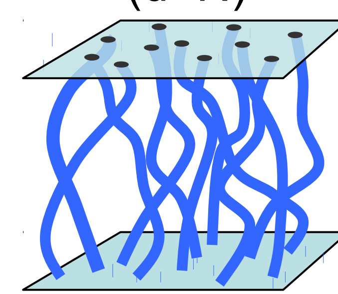


(1+2)



...

(d+N)



vortices (3+2)

Rough interfaces
and
Elastic lines in disordered systems

- Geometrical properties
  - Fluctuations: Roughness
  - Family-Vicsek scaling

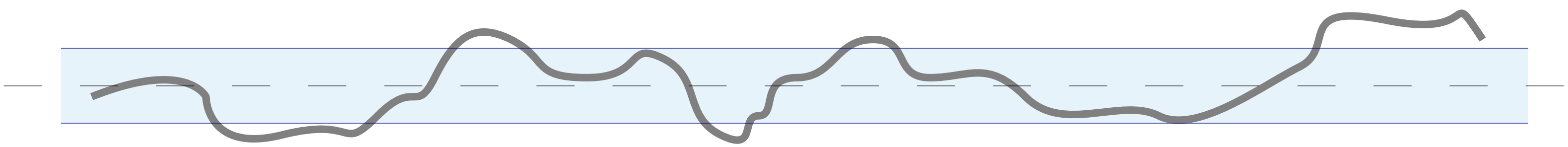
- Continuum equations
  - Edwards-Wilkinson
  - Kardar-Parisi-Zhang
  - Universality

- More on geometrical properties
  - Correlation functions
  - Anomalous scaling

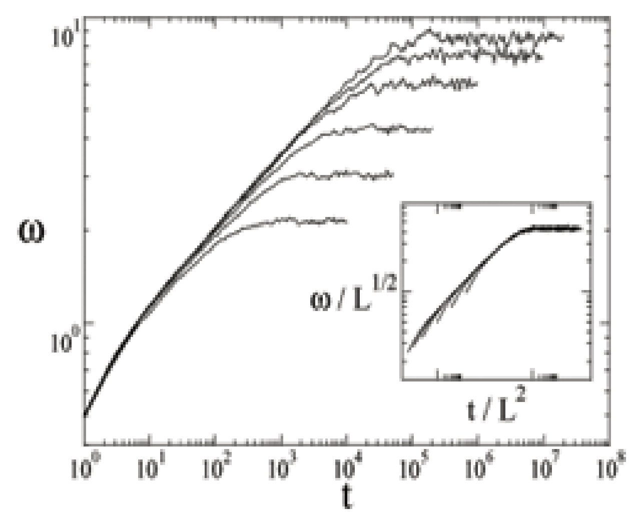
- Quenched disorder
  - Quenched disorder
  - Directed polymer
  - Thermal effects
  - Depinning transition
  - Avalanches

Correlation functions

global roughness $$W^2 = \sum_{z=0}^{L-1} \left[u(z) - \langle u(z)\rangle\right]^2 \qquad W(t,L) \sim \begin{cases} t^{\beta} & \text{for } \xi(t) \ll L \\ L^{\alpha} & \text{for } \xi(t) \gg L \end{cases}$$



automata model within de EW universality class

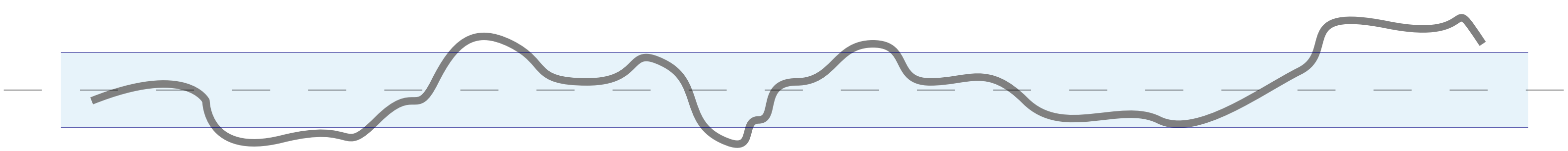


Mattos, Moreira, Atman, Brazilian J. Phys. **36**, 2006

Correlation functions

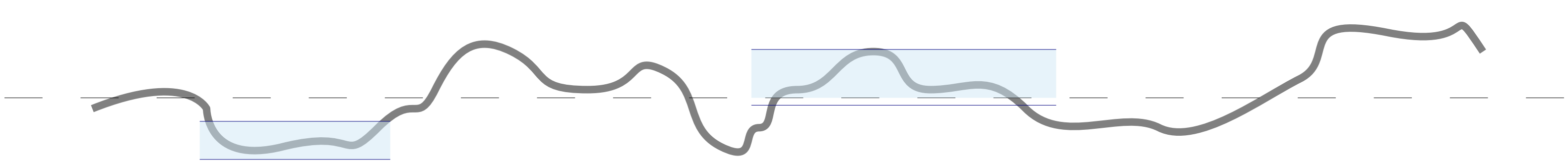
global roughness

$$W^2 = \sum_{z=0}^{L-1} [u(z) - \langle u(z) \rangle]^2 \qquad W(t,L) \sim \begin{cases} t^\beta & \text{for } \xi(t) \ll L \\ L^\alpha & \text{for } \xi(t) \gg L \end{cases}$$



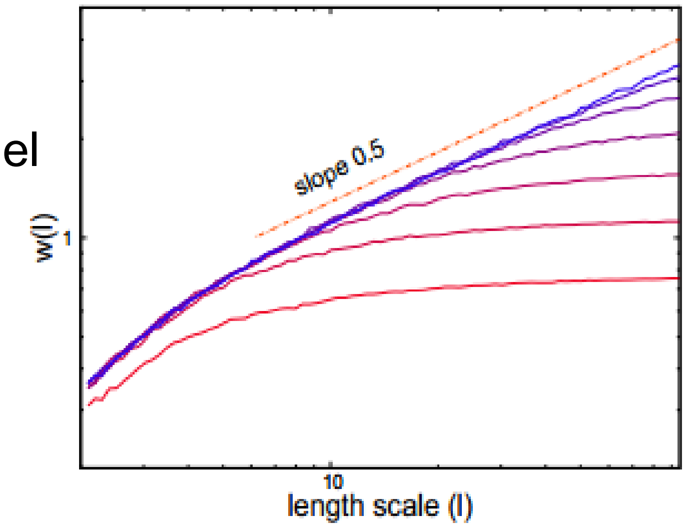
local roughness

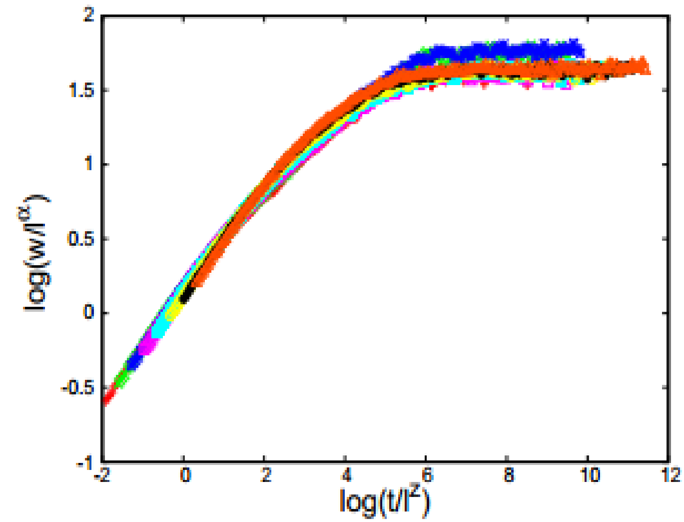
$$w(r,t)^2 = \sum_{r'} \frac{1}{r} \sum_{z=r'}^{r'+r} [u(z,t) - \langle u(z,t) \rangle_r]^2 \qquad w(r,t) \sim \begin{cases} t^\beta & \text{for } \xi(t) \ll r \\ r^\alpha & \text{for } \xi(t) \gg r \end{cases}$$



Correlation functions

intrinsic geometrical model with normal growth

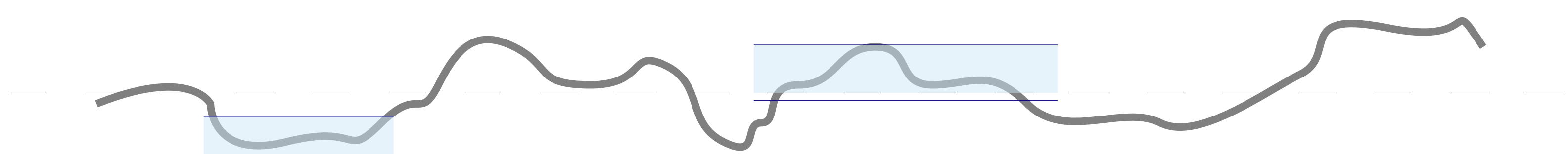




Rodríguez-Laguna, Santalla, Cuerno, JSTST, **P05032**, 2011

local roughness

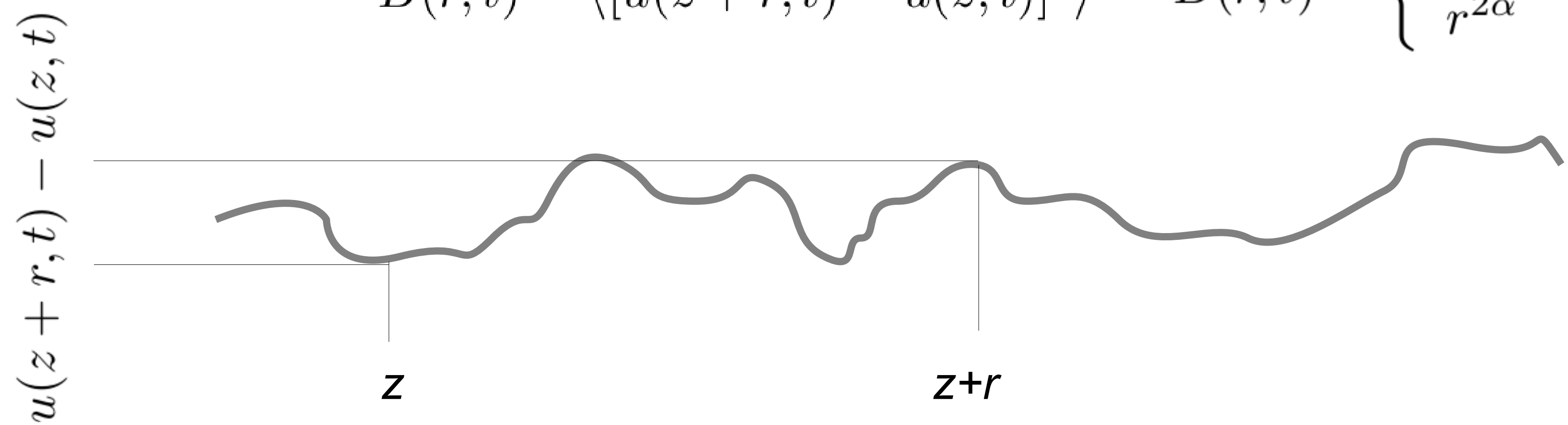
$$w(r,t)^2 = \sum_{r'} \frac{1}{r} \sum_{z=r'}^{r'+r} \left[u(z,t) - \langle u(z,t) \rangle_r\right]^2 \qquad w(r,t) \sim \begin{cases} t^\beta & \text{for } \xi(t) \ll r \\ r^\alpha & \text{for } \xi(t) \gg r \end{cases}$$

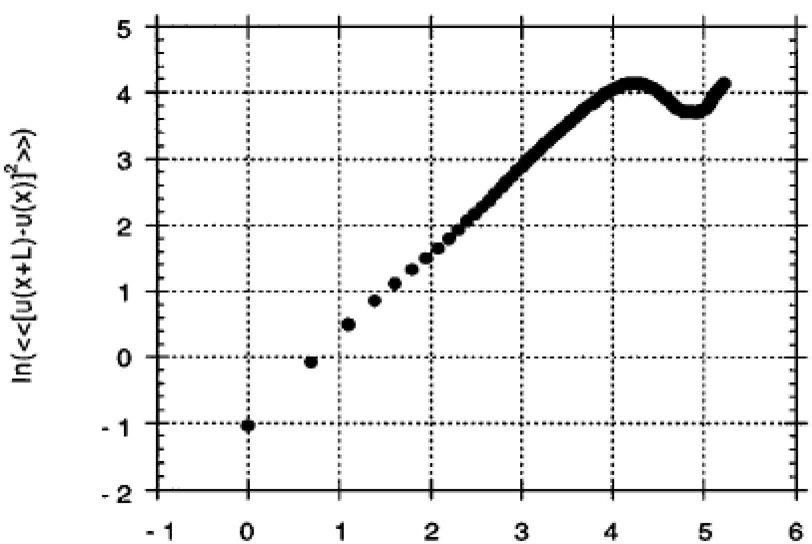


Correlation functions

displacement-displacement correlation function
(height-height correlation function)

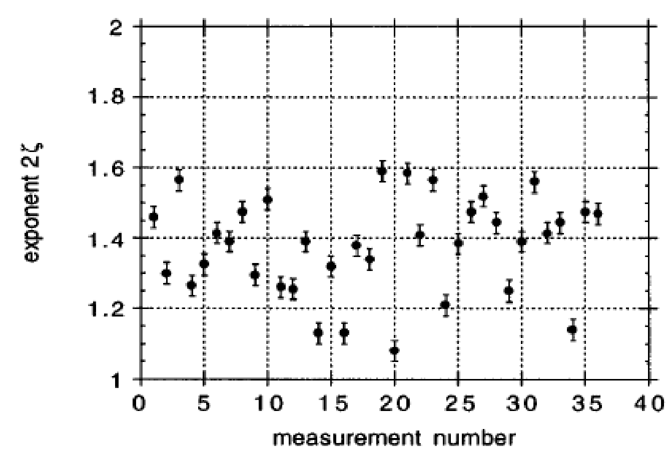
$$B(r,t) = \langle [u(z+r,t) - u(z,t)]^2 \rangle \qquad B(r,t) \sim \begin{cases} t^{2\beta} & \text{for } \xi(t) \ll r \\ r^{2\alpha} & \text{for } \xi(t) \gg r \end{cases}$$





Lemerle et al, PRL, **80**, 849 1998

experiments on ferromagnetic
domain wall motion



Correlation functions

structure factor

$$S(q,t) = \langle u(q,t)u(-q,t)\rangle \quad \text{with} \quad u(q,t) = \int dz\, u(z,t)\, e^{-iqz}$$

$$B(r,t) = \int \frac{dq}{\pi}\, [1-\cos(qr)]\; S(q)$$

$$S(q,t) \sim \begin{cases} t^{(1+2\alpha)/z} & \text{for } q \ll \xi(t)^{-1} \\ q^{-(1+2\alpha)} & \text{for } q \gg \xi(t)^{-1} \end{cases}$$

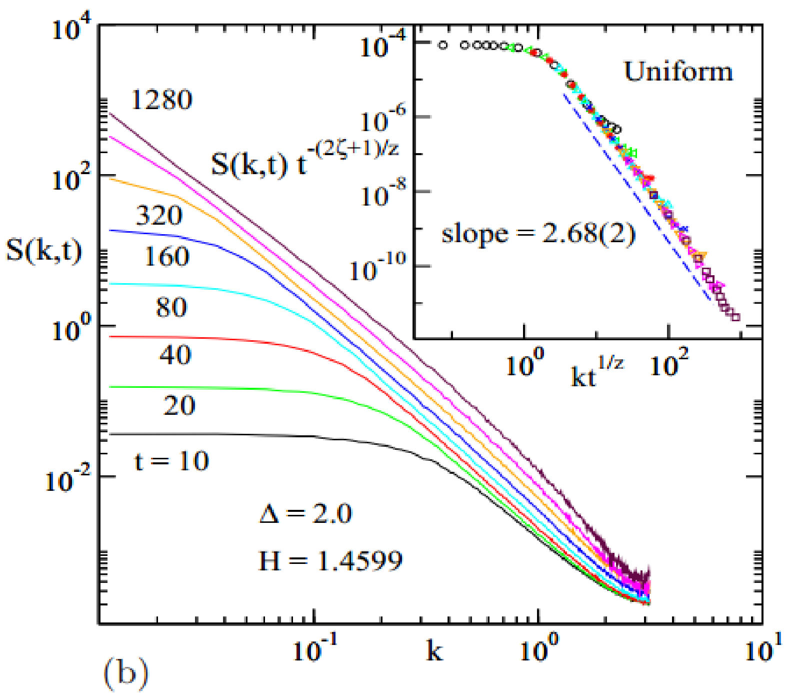
small q → large r

large q → small r

Correlation functions

structure factor

$$S(q,t) = \langle u(q,t)u(-q,t)\rangle \quad \text{with} \quad u(q,t) = \int dz\, u(z,t)\, e^{-iqz}$$



$$S(q,t) \sim \begin{cases} t^{(1+2\alpha)/z} & \text{for } q \ll \xi(t)^{-1} \\ q^{-(1+2\alpha)} & \text{for } q \gg \xi(t)^{-1} \end{cases}$$

small q → large r

large q → small r

depinning of the Random field Ising model

Quin, Zheng, Zhou, JPA, **45**, 115001, 2012

Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

α : global roughness exponent
α_{loc} : local roughness exponent
α_S : spectral roughness exponent

$$B(r,t) = \int \frac{dq}{\pi} \left[1 - \cos(qr)\right] S(q)$$

The convergence of the integral depends on the value of α_S

$$B(r) \sim r^{2\alpha_{loc}} \sim \begin{cases} r^{2\alpha_S} & \text{for } \alpha_S < 1 \\ r^2 & \text{for } \alpha_S > 1 \end{cases}$$

$\alpha_{\text{loc}} = \alpha_s$ intrinsic anomalous scaling
$\alpha_{\text{loc}} = 1$ super rough anomalous scaling

α ?

López, Rodriguez, Cuerno, PRE, **56**, 3993, 1997

Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

General classification for anomalous scaling

$$\begin{cases} \text{if } \alpha_s < 1 \Rightarrow \alpha_{\text{loc}} = \alpha_s & \begin{cases} \alpha_s = \alpha \Rightarrow & \text{Family-Vicsek} \\ \alpha_s \neq \alpha \Rightarrow & \text{intrinsic} \end{cases} \\ \text{if } \alpha_s > 1 \Rightarrow \alpha_{\text{loc}} = 1 & \begin{cases} \alpha_s = \alpha \Rightarrow & \text{super rough} \\ \alpha_s \neq \alpha \Rightarrow & \text{faceted} \end{cases} \end{cases}$$

Ramasco, López, Rodriguez, PRL, **84**, 2199, 2000

Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

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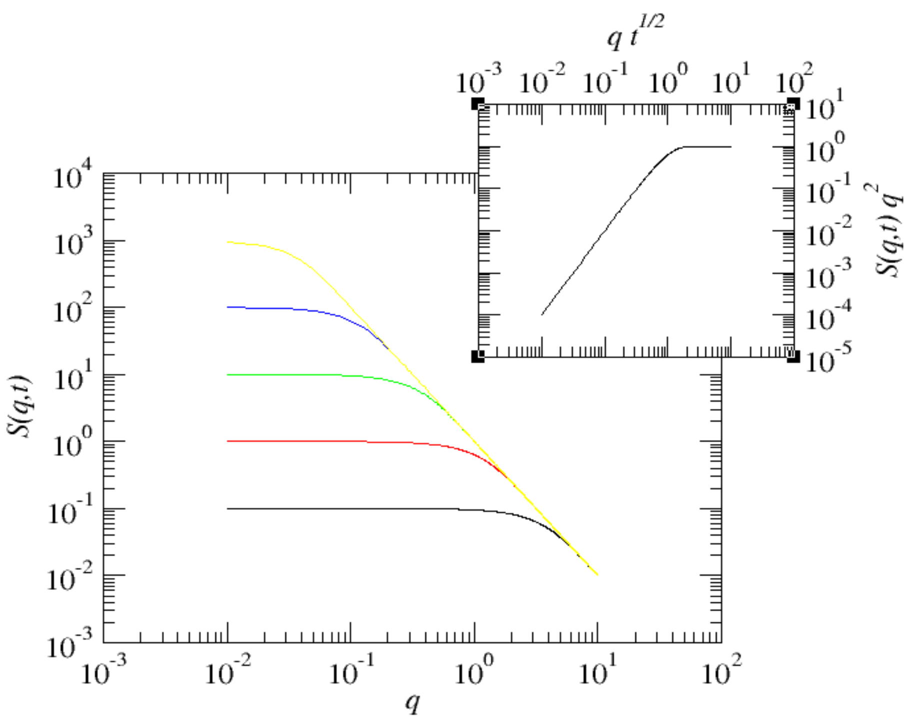

Edwards-Wilkinson equation

$$S(q,t) \sim \begin{cases} t^{(1+2\alpha)/z} & \text{for } q \ll \xi(t)^{-1} \\ q^{-(1+2\alpha)} & \text{for } q \gg \xi(t)^{-1} \end{cases}$$

Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

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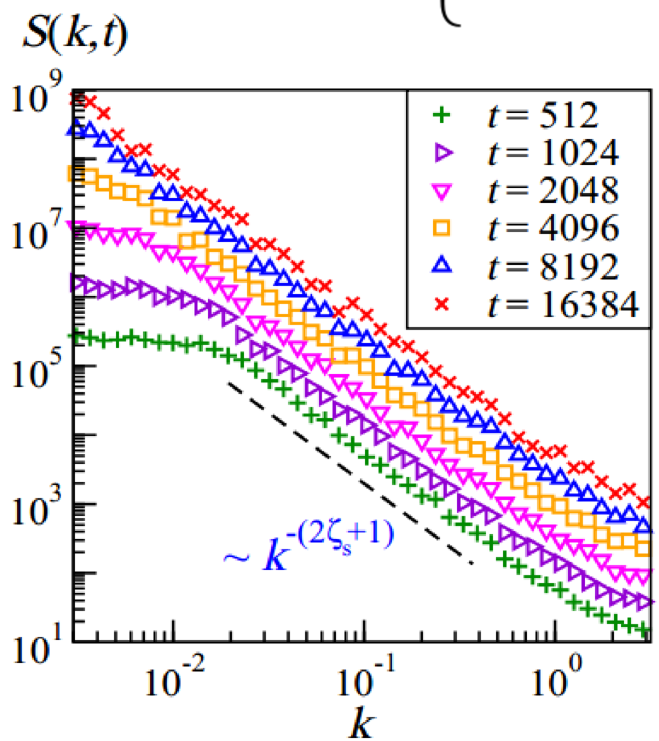


$$\alpha_S = 0.5$$

ferromagnetic thin film model

$$S(q,t) \sim \begin{cases} t^{(1+2\alpha)/z} & \text{for } qt^{1/z} \ll 1 \\ q^{-(1+2\alpha_S)} t^{2(\alpha-\alpha_S)/z} & \text{for } qt^{1/z} \gg 1 \end{cases}$$

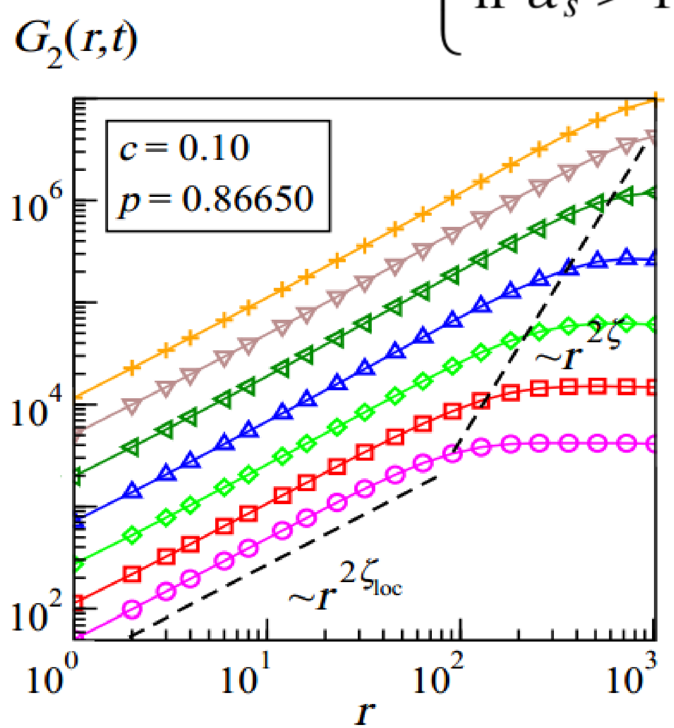
Torres, Buceta, EPJB, **86**, 20, 2013

Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

General classification for anomalous scaling

$$\begin{cases} \text{if } \alpha_s < 1 \Rightarrow \alpha_{\text{loc}} = \alpha_s & \begin{cases} \alpha_s = \alpha \Rightarrow & \text{Family-Vicsek} \\ \alpha_s \neq \alpha \Rightarrow & \textbf{intrinsic} \end{cases} \\ \text{if } \alpha_s > 1 \Rightarrow \alpha_{\text{loc}} = 1 & \begin{cases} \alpha_s = \alpha \Rightarrow & \text{super rough} \\ \alpha_s \neq \alpha \Rightarrow & \text{faceted} \end{cases} \end{cases}$$



$$\alpha_S = 0.5 \quad \alpha_{loc} = 0.5$$

$$B(r,t) \sim \begin{cases} r^{2\alpha_{loc}} t^{2(\alpha-\alpha_{loc})/z} & \text{for } rt^{-1/z} \ll 1 \\ t^{2\alpha/z} & \text{for } rt^{-1/z} \gg 1 \end{cases}$$

Torres, Buceta, EPJB, **86**, 20, 2013

Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

General classification for anomalous scaling

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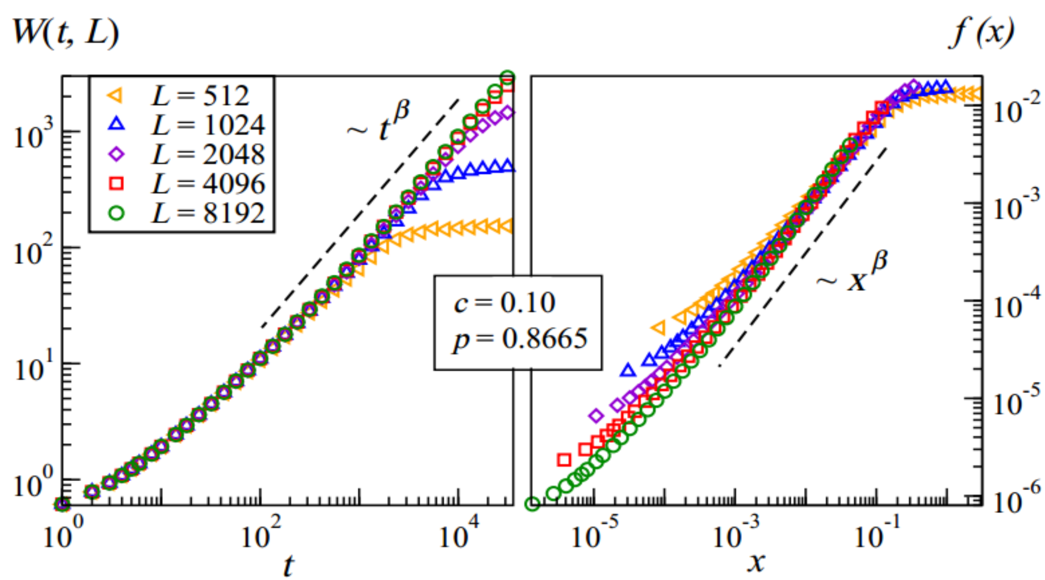
$$\alpha_S = 0.5 \quad \alpha_{loc} = 0.5 \quad \alpha = 1.5$$



Torres, Buceta, EPJB, **86**, 20, 2013

Anomalous scaling

$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

General classification for anomalous scaling

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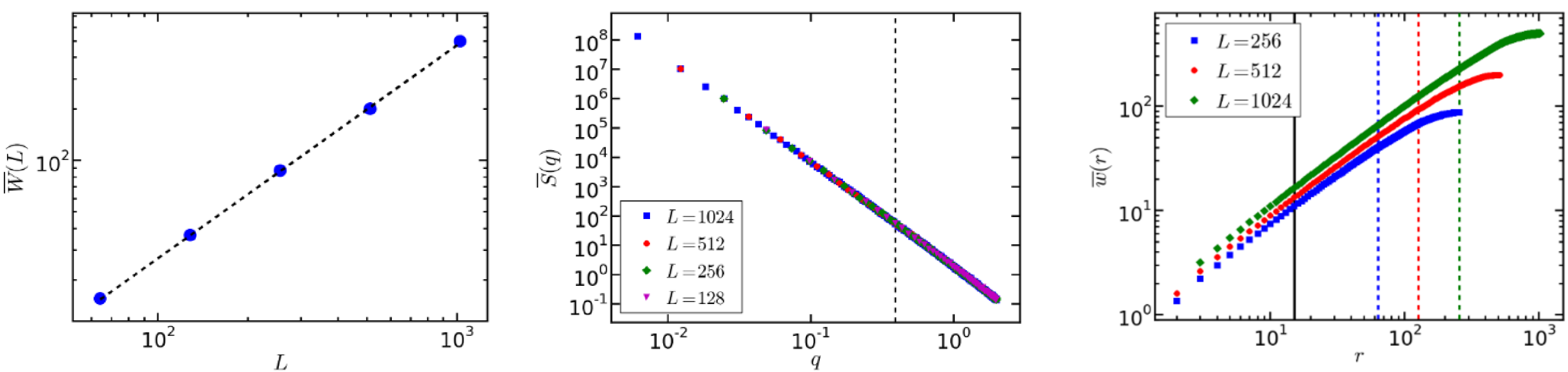
interface at critical depinning driven quenched EW equation

$$\alpha_S = 1.25 \qquad \alpha_{loc} = 1 \qquad \alpha = 1.25$$



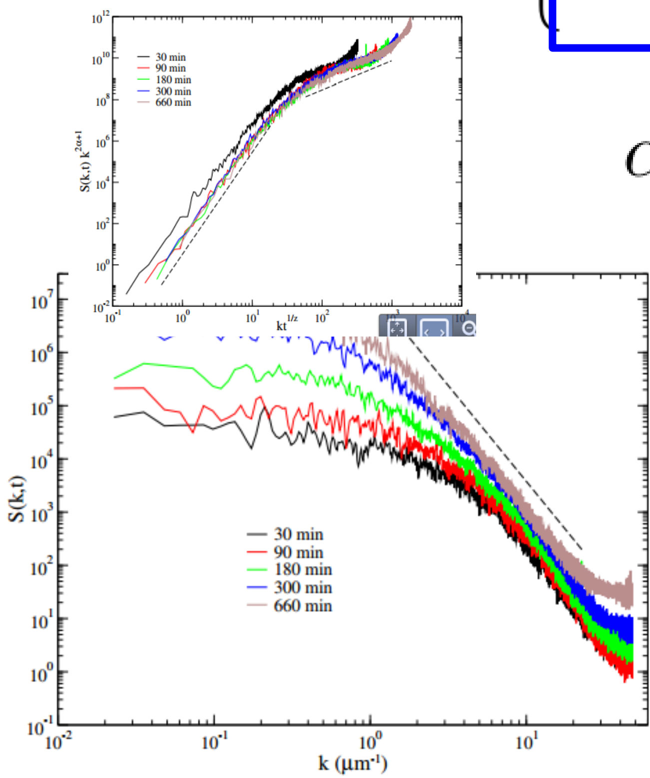
Anomalous scaling

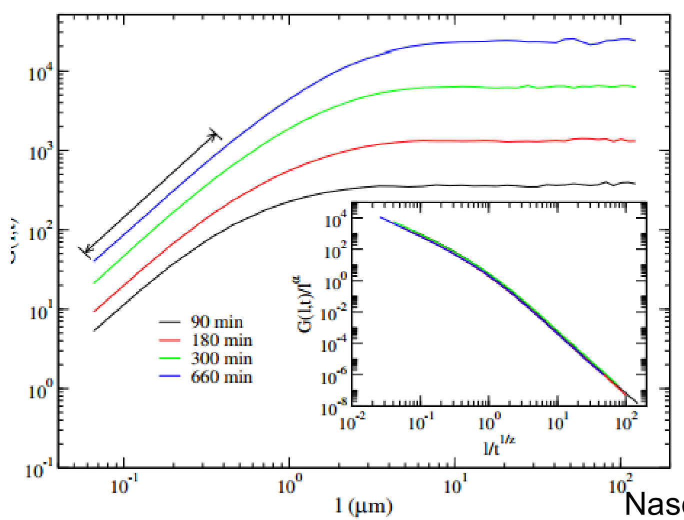
$$W(L) \sim L^{\alpha} \qquad w(r) \sim r^{\alpha_{loc}} \qquad B(r) \sim r^{2\alpha_{loc}} \qquad S(q) \sim q^{-(1+2\alpha_S)}$$

General classification for anomalous scaling

$$\begin{cases} \text{if } \alpha_s < 1 \Rightarrow \alpha_{\text{loc}} = \alpha_s & \begin{cases} \alpha_s = \alpha \Rightarrow & \text{Family-Vicsek} \\ \alpha_s \neq \alpha \Rightarrow & \text{intrinsic} \end{cases} \\ \text{if } \alpha_s > 1 \Rightarrow \alpha_{\text{loc}} = 1 & \begin{cases} \alpha_s = \alpha \Rightarrow & \text{super rough} \\ \alpha_s \neq \alpha \Rightarrow & \textbf{faceted} \end{cases} \end{cases}$$

$$\alpha_S = 1.29 \quad \alpha_{loc} = 0.82 \quad \alpha = 1.94$$





epitaxial growth of semiconductor films (2+1)

Nascimiento, Ferreira, Ferreira,EPL, **94**, 68002, 2011

Rough interfaces
and
Elastic lines in disordered systems

- Geometrical properties
  - Fluctuations: Roughness
  - Family-Vicsek scaling

- Continuum equations
  - Edwards-Wilkinson
  - Kardar-Parisi-Zhang
  - Universality

- More on geometrical properties
  - Correlation functions
  - Anomalous scaling

- Quenched disorder
  - Quenched disorder
  - Directed polymer
  - Thermal effects
  - Depinning transition
  - Avalanches

Quenched disorder

Basic dynamic equation: EW + KPZ + force + temperature + quenched noise

$$\partial_t u = \nu \partial_z^2 u + \frac{\lambda}{2} \left(\partial_z u\right)^2 + F + \eta(z,t) + \xi(u,z)$$

$$\begin{aligned} \langle \eta(z,t) \rangle &= 0 \\ \langle \eta(z,t)\eta(z',t') \rangle &= \frac{2T}{\gamma}\delta(z-z')\delta(t-t') \end{aligned}$$

Quenched disorder

Basic dynamic equation: EW + KPZ + force + temperature + quenched noise

$$\partial_t u = \nu \partial_z^2 u + \frac{\lambda}{2}(\partial_z u)^2 + F + \eta(z,t) + \xi(u,z)$$

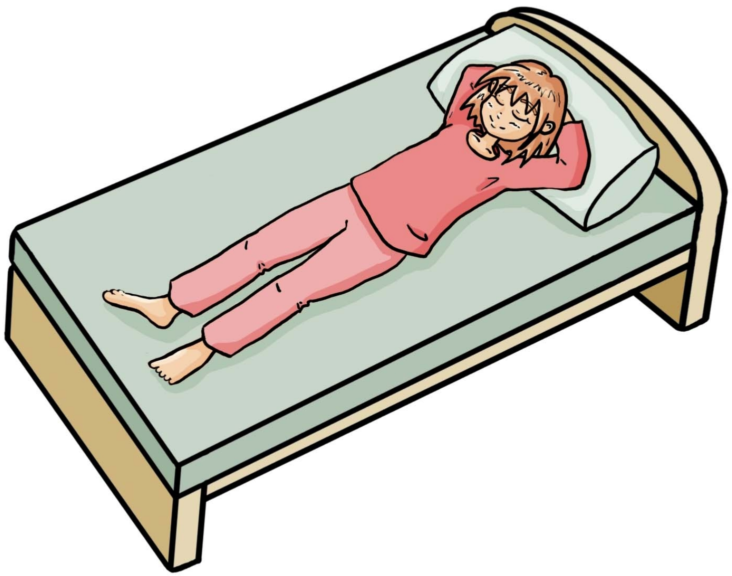
What is the basic effect of the quenched disorder?

$$\boxed{\partial_t u = \nu \partial_z^2 u + \xi(u,z)}$$

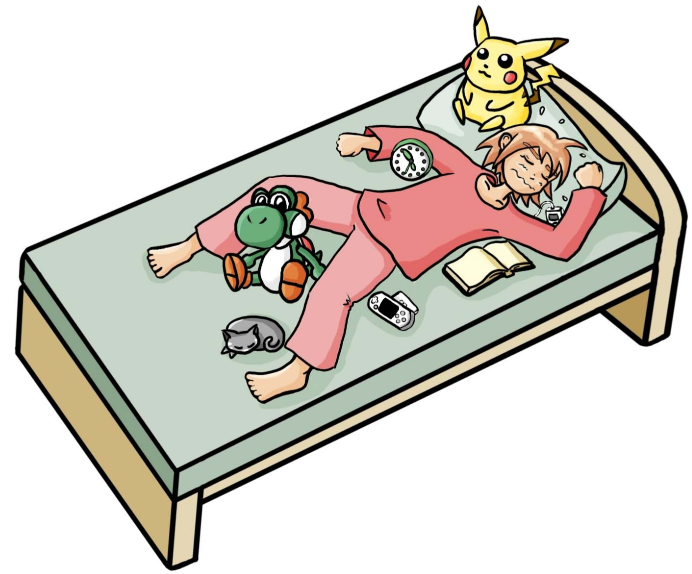
competition between elasticity and disorder

Quenched disorder

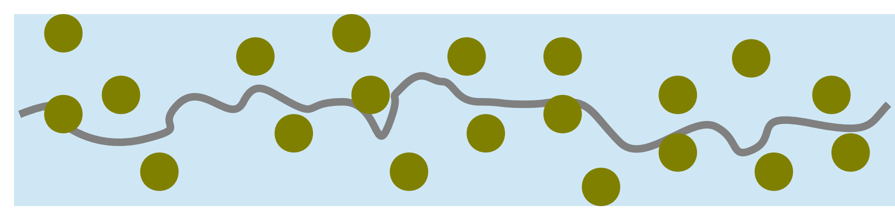
elasticity



disorder



the interface tends to be **flat**



the interface tends to be **rough**

the system minimizes the energy by pinning to impurities

Quenched disorder

$$\partial_t u = \nu \partial_z^2 u + \xi(u,z)$$

Hamiltonian: $$\mathcal{H} = \int_L dz \left[\frac{c}{2} \left(\frac{\partial u}{\partial z} \right)^2 + V(u,x) \right]$$

$V(u,x)$: disorder potential $$\xi(u,z) = -\frac{\partial V(u,z)}{\partial u}$$

Random field

$$\begin{aligned} \overline{\xi(u,z)} &= 0 \\ \overline{\xi(u,z)\xi(u',z')} &= D_{\mathrm{RF}}\delta(u-u')\delta(z-z') \end{aligned}$$

$$V(u,z) = \int_{-\infty}^{u} du' \; \xi(u',x)$$

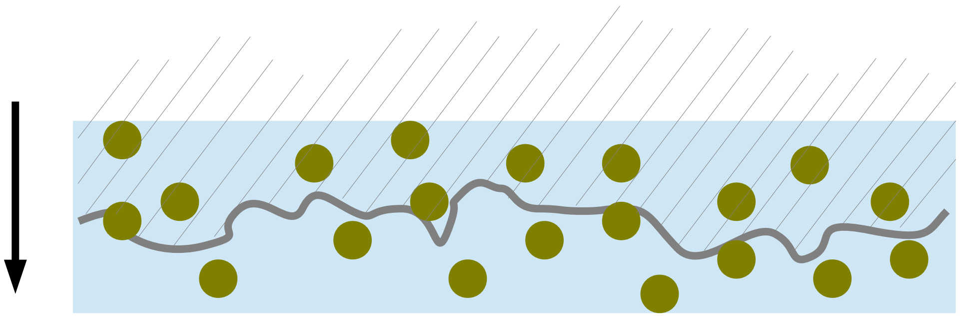
the interface has a memory of the disorder left behind

Random bond

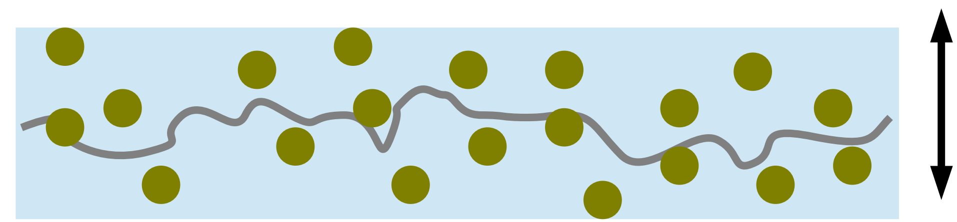
$$\begin{aligned} \overline{V(u,z)} &= 0 \\ \overline{V(u,z)V(u',z')} &= D_{\mathrm{RB}}\delta(u-u')\delta(z-z') \end{aligned}$$

the interface explores the disorder point by point

Quenched disorder



up-down symmetry broken



Random field

$$\overline{\xi(u,z)} = 0$$
$$\overline{\xi(u,z)\xi(u',z')} = D_{\mathrm{RF}}\delta(u-u')\delta(z-z')$$

$$V(u,z) = \int_{-\infty}^{u} du'\; \xi(u',x)$$

the interface has a memory of the disorder left behind

Random bond

$$\overline{V(u,z)} = 0$$
$$\overline{V(u,z)V(u',z')} = D_{\mathrm{RB}}\delta(u-u')\delta(z-z')$$

the interface explores the disorder point by point

Quenched disorder

Random bond

$$\mathcal{H} = \int_L dz \left[\frac{c}{2} \left(\frac{\partial u}{\partial z} \right)^2 + V(u, x) \right]$$

$$z \to z' = bz$$

$$u \to u' = b^\alpha u$$

$$\mathcal{H}_{\rm el} \quad \to \quad \int b\, dz \frac{b^{2\alpha}}{b^2} \left(\frac{\partial u}{\partial z} \right)^2 \sim b^{2\alpha-1} \mathcal{H}_{\rm el}$$

$$\mathcal{H}_{\rm dis} \quad \to \quad \int b\, dz b^{-\alpha/2} b^{-1/2} V(u, z) \sim b^{(1-\alpha)/2} \mathcal{H}_{\rm dis}$$

$$\mathcal{H} \quad \to \quad b^{2\alpha-1} \left(\mathcal{H}_{\rm el} + \frac{b^{(1-\alpha)/2}}{b^{2\alpha-1}} \mathcal{H}_{\rm dis} \right)$$

$$\mathcal{H} \quad \to \quad b^{2\alpha-1} \mathcal{H} \quad \longrightarrow \quad \boxed{\alpha_{\rm RM} = \alpha_{\rm F} = 3/5}$$

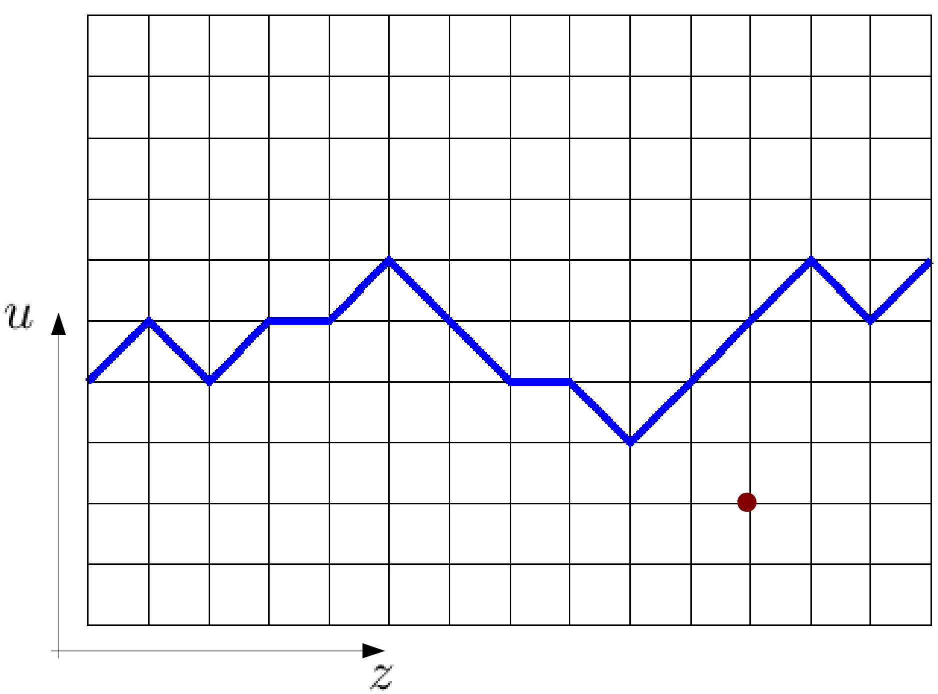
Random Manifold
Flory exponent
(for the RB case)

the roughness exponent is larger than with pure elasticity $\alpha_{\rm RM} > \alpha_{\rm EW}$

disorder roughens the interface

but the exponent is wrong!!!

Directed polymer



solid-on-solid condition

$$|u(z+1) - u(z)| = 0, 1$$

effective elasticity – correlations

“elastic” energy cost

$$E\left[|u(z+1) - u(z)| = 1\right] = \nu$$

quenched disordered potential

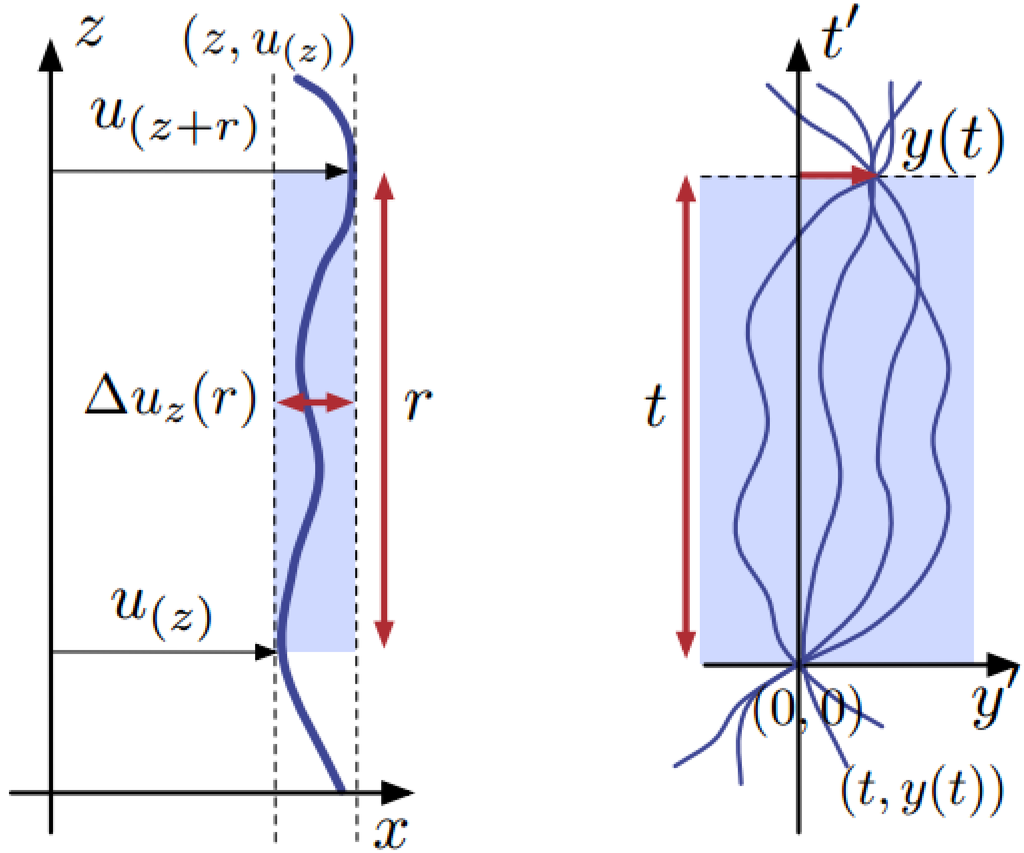
$$\varepsilon(u, z)$$

if $\varepsilon(u, z) = \varepsilon_0$ then we have a random walk with $W^2 \sim r^{2\alpha} \sim r \sim r^{2\alpha_{\mathrm{EW}}}$

(the EW roughness exponent is equivalent to the random-walk normal-diffusion exponent)

if $\varepsilon(u, z)$ is random, then we expect $W^2 \sim r^{2\alpha_{\mathrm{RM}}}$

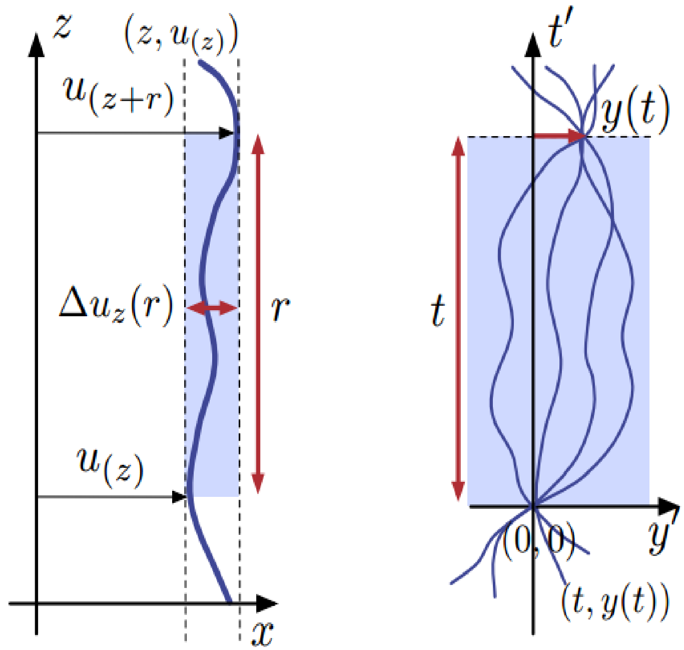
Directed polymer



$$u(z) \leftrightarrow y(t)$$

Directed polymer

$$u(z) \leftrightarrow y(t)$$



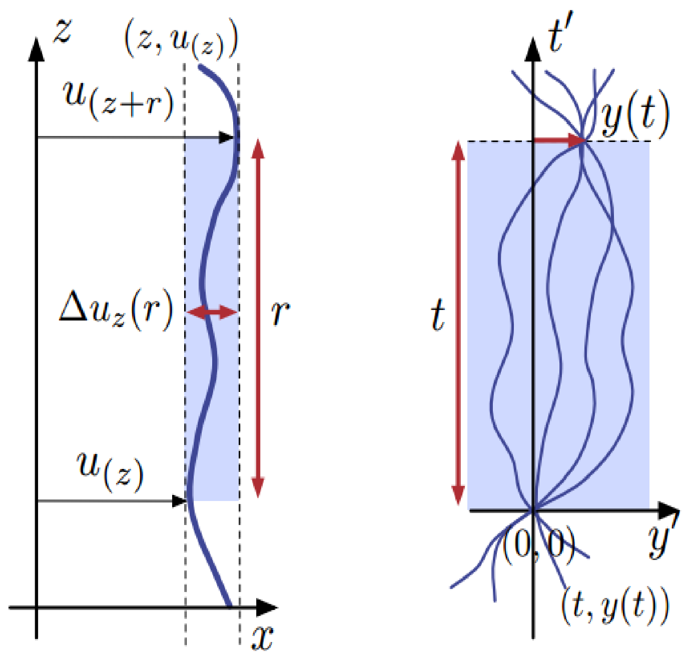
$$\mathcal{H}_V[y(t)] = \int_0^t dt' \left[\frac{\nu}{2} \left(\frac{\partial y}{\partial t'} \right)^2 + V(y(t'), t') \right]$$

diffusive term

$$\frac{\partial y(t)}{\partial t} = \nu \frac{\partial^2 y(t)}{\partial t^2} + \xi(y, t)$$

Directed polymer

$$u(z) \leftrightarrow y(t)$$



$$\mathcal{H}_V[y(t)] = \int_0^t dt' \left[\frac{\nu}{2} \left(\frac{\partial y}{\partial t'} \right)^2 + V(y(t'), t') \right]$$

$$P_V[y(t)] = \frac{e^{-\beta \mathcal{H}[y(t)]}}{\int \mathcal{D}y(t) e^{-\beta \mathcal{H}[y(t)]}} = \frac{e^{-\beta \mathcal{H}[y(t)]}}{\mathcal{Z}_V[y(t)]}$$

disroder average

$$P[y(t)] = \overline{P_V[y(t)]}$$

directed polymer partition function

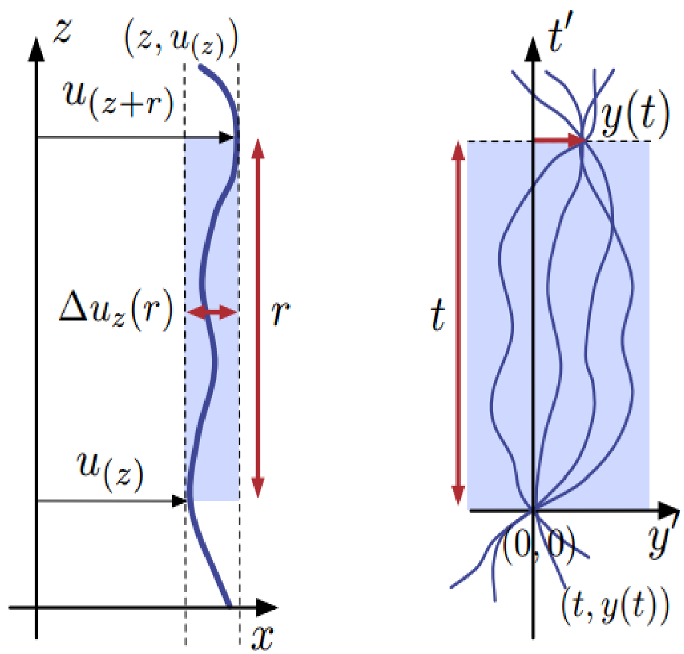
$$\mathcal{Z}_V[y(t)] = \int_{(0,0)}^{y(t)} \mathcal{D}y(t) \exp\left\{ -\beta \int dt' \left[\frac{\nu}{2} \left(\frac{\partial y}{\partial t'} \right)^2 + V(y(t'), t') \right] \right\}$$

the partition function obeys the Cole-Hopf equation
(diffusion equation with multiplicative noise)

$$\frac{\partial \mathcal{Z}_V[y(t)]}{\partial t} = \frac{1}{2\nu\beta} \frac{\partial^2 \mathcal{Z}_V[y(t)]}{\partial y^2} - \beta \mathcal{Z}_V[y(t)] V(y(t), t)$$

Directed polymer

$$u(z) \leftrightarrow y(t)$$



$$\mathcal{Z}_V[y(t)] = \int_{(0,0)}^{y(t)} \mathcal{D}y(t) e^{\mathcal{H}_V[y(t)]} = e^{-\beta \mathcal{F}_V[y(t)]}$$

$$\mathcal{F}_V[y(t)] = -\frac{1}{\beta} \ln \mathcal{Z}_V[y(t)]$$

Hopf transformation

directed polymer free energy

Cole-Hopf equation

$$\frac{\partial \mathcal{Z}_V[y(t)]}{\partial t} = \frac{1}{2\nu\beta} \frac{\partial^2 \mathcal{Z}_V[y(t)]}{\partial y^2} - \beta \mathcal{Z}_V[y(t)] V(y(t), t)$$

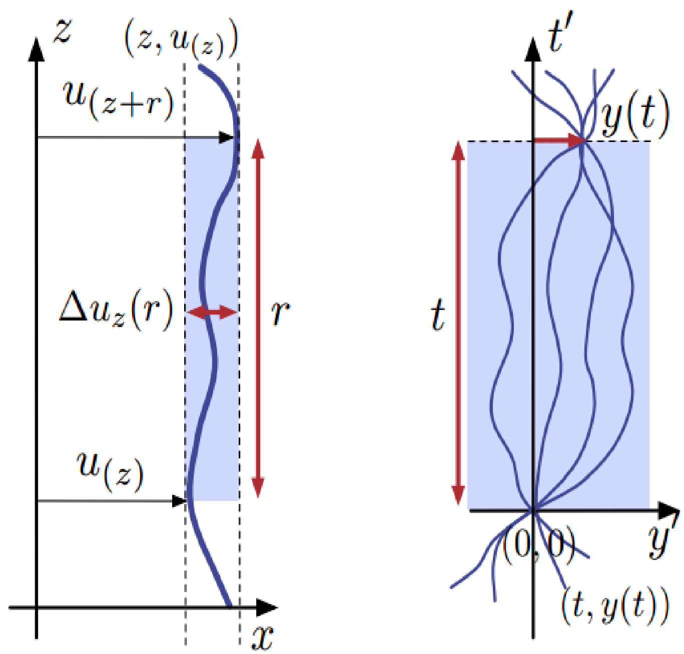
free energy evolution equation

$$\frac{\partial \mathcal{F}_V[y(t)]}{\partial t} = \frac{1}{2\nu\beta} \frac{\partial^2 \mathcal{F}_V[y(t)]}{\partial y^2} + \frac{1}{2\nu} \left(\frac{\partial \mathcal{F}_V[y(t)]}{\partial y} \right)^2 + V(y, t)$$

Quenched disorder

Directed polymer

$$u(z) \leftrightarrow y(t)$$



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$$\mathcal{F}_V[y(t)] = -\frac{1}{\beta} \ln \mathcal{Z}_V[y(t)]$$

Hopf transformation

directed polymer free energy

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} = \nu' \frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2} + \frac{\lambda'}{2} \left(\frac{\partial \tilde{u}}{\partial \tilde{z}} \right)^2 + \eta(\tilde{z}, \tilde{t})$$

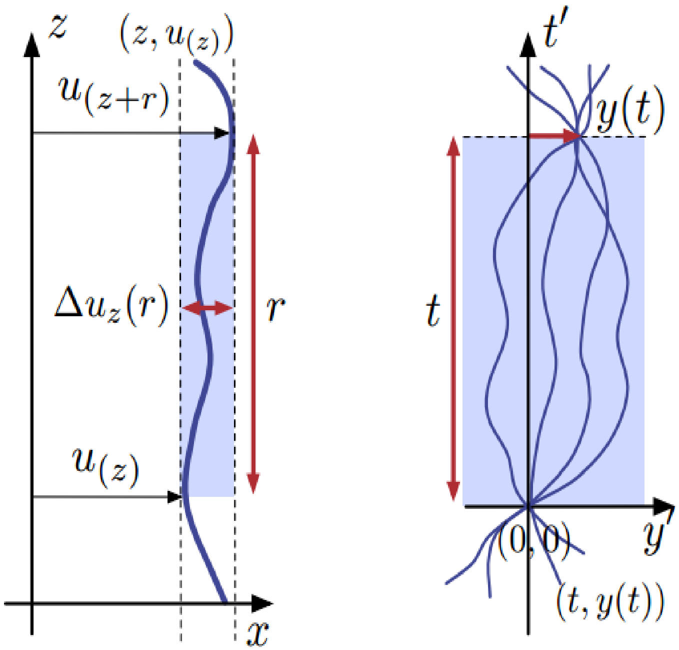
KPZ

free energy evolution equation

$$\frac{\partial \mathcal{F}_V[y(t)]}{\partial t} = \frac{1}{2\nu\beta} \frac{\partial^2 \mathcal{F}_V[y(t)]}{\partial y^2} + \frac{1}{2\nu} \left(\frac{\partial \mathcal{F}_V[y(t)]}{\partial y} \right)^2 + V(y,t)$$

Directed polymer

$$u(z) \leftrightarrow y(t)$$



KPZ		DP		RM
$\tilde{u}$	$\rightarrow$	$\mathcal{F}$	$\rightarrow$	E
$\tilde{z}$	$\rightarrow$	y	$\rightarrow$	u
$\tilde{t}$	$\rightarrow$	t	$\rightarrow$	z

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} = \nu' \frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2} + \frac{\lambda'}{2} \left(\frac{\partial \tilde{u}}{\partial \tilde{z}} \right)^2 + \eta(\tilde{z}, \tilde{t})$$

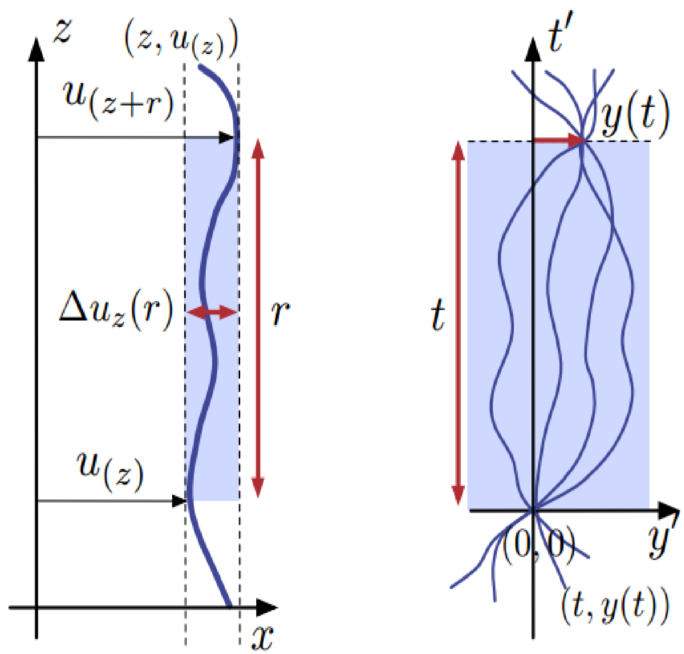
KPZ

free energy evolution equation

$$\frac{\partial \mathcal{F}_V[y(t)]}{\partial t} = \frac{1}{2\nu\beta} \frac{\partial^2 \mathcal{F}_V[y(t)]}{\partial y^2} + \frac{1}{2\nu} \left(\frac{\partial \mathcal{F}_V[y(t)]}{\partial y} \right)^2 + V(y,t)$$

Directed polymer

$$u(z) \leftrightarrow y(t)$$



KPZ		DP		RM
$\tilde{u}$	$\to$	$\mathcal{F}$	$\to$	E
$\tilde{z}$	$\to$	y	$\to$	u
$\tilde{t}$	$\to$	t	$\to$	z

$$\tilde{z} \sim \tilde{t}^{1/z_{\rm KPZ}} \quad \to \quad y \sim t^{1/z_{\rm KPZ}} \quad \to \quad u \sim z^{\alpha_{\rm RM}}$$

$$\tilde{u} \sim \tilde{t}^{\beta_{\rm KPZ}} \quad \to \quad \mathcal{F} \sim t^{\beta_{\rm KPZ}} \quad \to \quad E \sim z^{\varpi_{\rm RM}}$$

$$\alpha_{\rm RM} = 1/z_{\rm KPZ}$$

$$\varpi_{\rm RM} = \beta_{\rm KPZ}$$

$$\alpha_{\rm RM} = 2/3$$

$$\alpha_{\rm RM} = 2/3 \neq \alpha_{\rm RM-Flory} = 3/5$$

$$\alpha_{\rm RM} = 2/3 > \alpha_{\rm EW} = 1/2$$

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} = \nu' \frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2} + \frac{\lambda'}{2}\left(\frac{\partial \tilde{u}}{\partial \tilde{z}}\right)^2 + \eta(\tilde{z}, \tilde{t})$$

KPZ

free energy evolution equation

$$\frac{\partial \mathcal{F}_V[y(t)]}{\partial t} = \frac{1}{2\nu\beta}\frac{\partial^2 \mathcal{F}_V[y(t)]}{\partial y^2} + \frac{1}{2\nu}\left(\frac{\partial \mathcal{F}_V[y(t)]}{\partial y}\right)^2 + V(y,t)$$

(KPZ equation → directed polymer → random manifold roughness exponent)

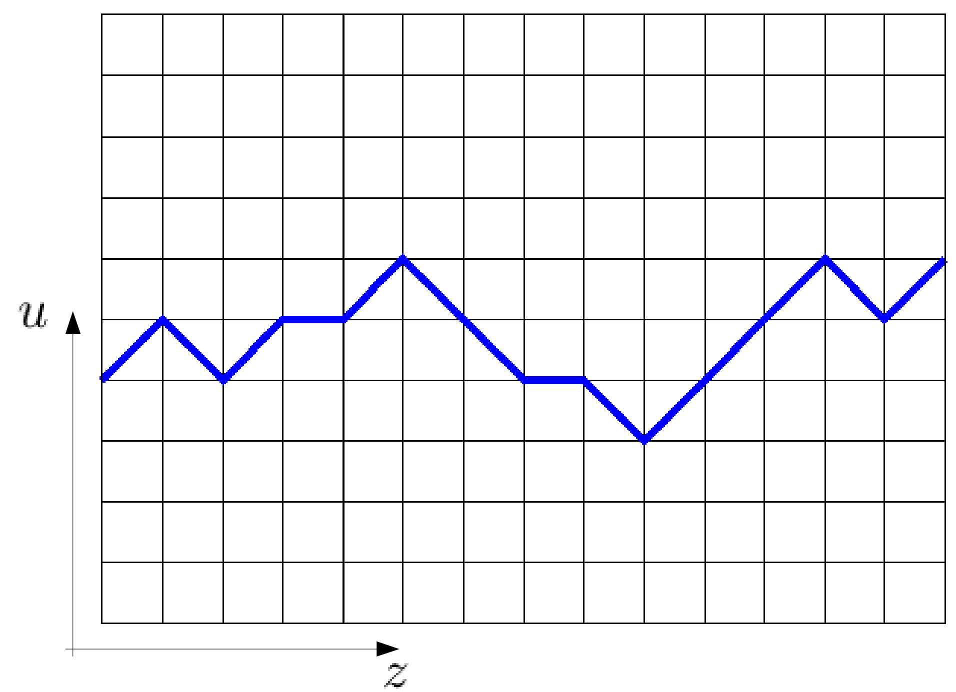
Directed polymer

Transfer matrix

$$\mathcal{Z} = \sum_{\{u(z)\}} e^{-\beta\mathcal{H}[u(z)]}$$

$$\mathcal{H}[u(z)] = \sum_z \varepsilon[u(z), z]$$

Problem: what is the minimum energy path of length z?



Directed polymer

Transfer matrix

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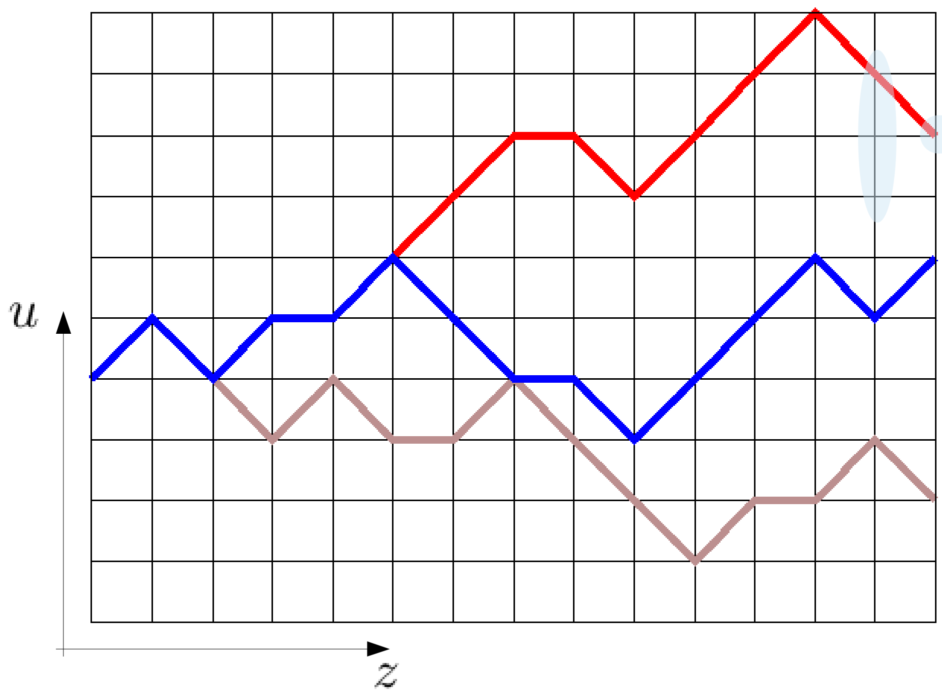


Problem: what is the minimum energy path of length z?

first, search the path arriving at (u, z) with minimum energy

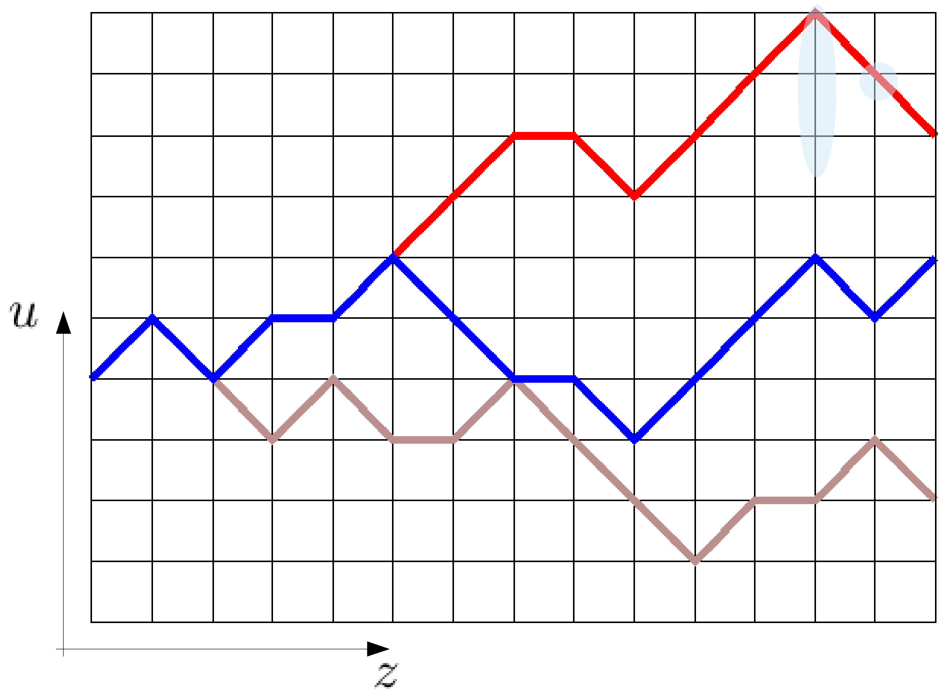
$$E(u, z) = \min[E(u-1, z-1), E(u, z-1), E(u+1, z-1)] + \varepsilon(u, z)$$

Directed polymer

Transfer matrix

$$\mathcal{Z} = \sum_{\{u(z)\}} e^{-\beta \mathcal{H}[u(z)]}$$

$$\mathcal{H}[u(z)] = \sum_z \varepsilon[u(z), z]$$



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Directed polymer

Transfer matrix

$$\mathcal{Z} = \sum_{\{u(z)\}} e^{-\beta \mathcal{H}[u(z)]}$$

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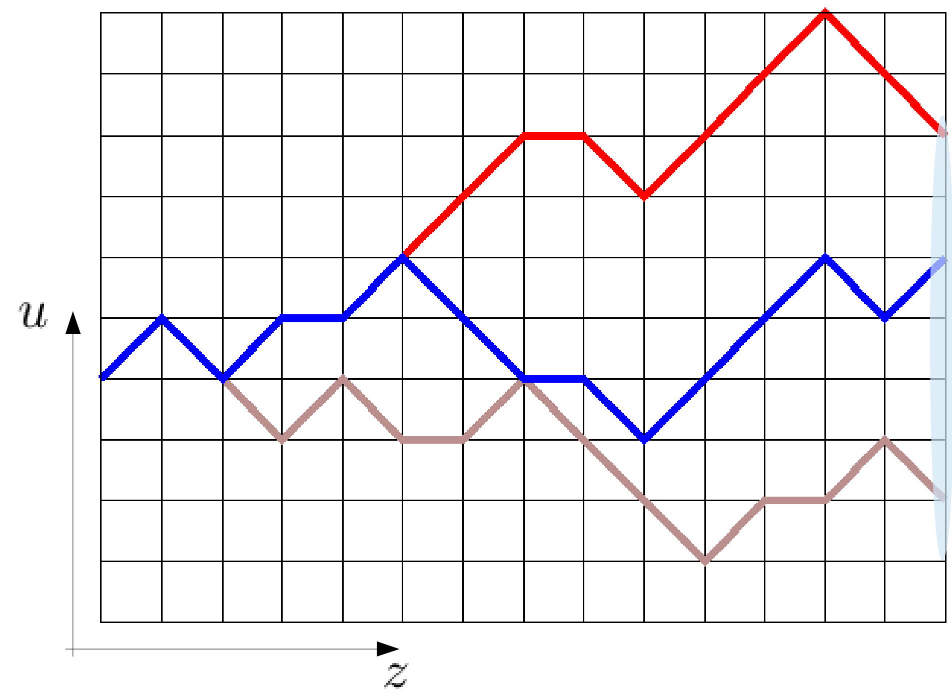


Problem: what is the minimum energy path of length z?

first, search the path arriving at (u, z) with minimum energy

$$E(u, z) = \min[E(u-1, z-1), E(u, z-1), E(u+1, z-1)] + \varepsilon(u, z)$$

then, keep only the minimun energy path

$$E(z) = \min_{\{z\}} E(u, z)$$

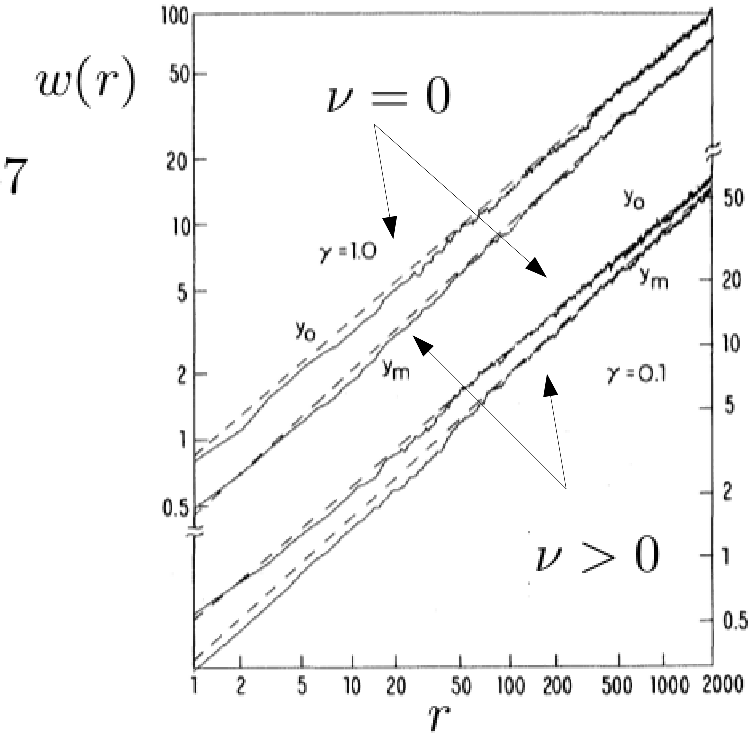
Directed polymer

Transfer matrix

$$\alpha(\nu = 0) = 0.62 \quad \alpha(\nu > 0) = 0.67$$

in agreement with

$$\alpha_{\rm RM} = 2/3$$



first, search the path arriving at (u, z) with minimum energy

$$E(u,z) = \min[E(u-1,z-1), E(u,z-1), E(u+1,z-1)] + \varepsilon(u,z)$$

then, keep only the minimun energy path

$$E(z) = \min_{\{z\}} E(u,z)$$

Kardar,PRL, **55**, 2923, 1985

Thermal effects

competition between thermal and disorder fluctuations

$$\partial_t u = \nu \partial_z^2 u + \eta(z,t) + \xi(u,z)$$

thermal effects dominates at short length scales, but disorder wins at large length scales

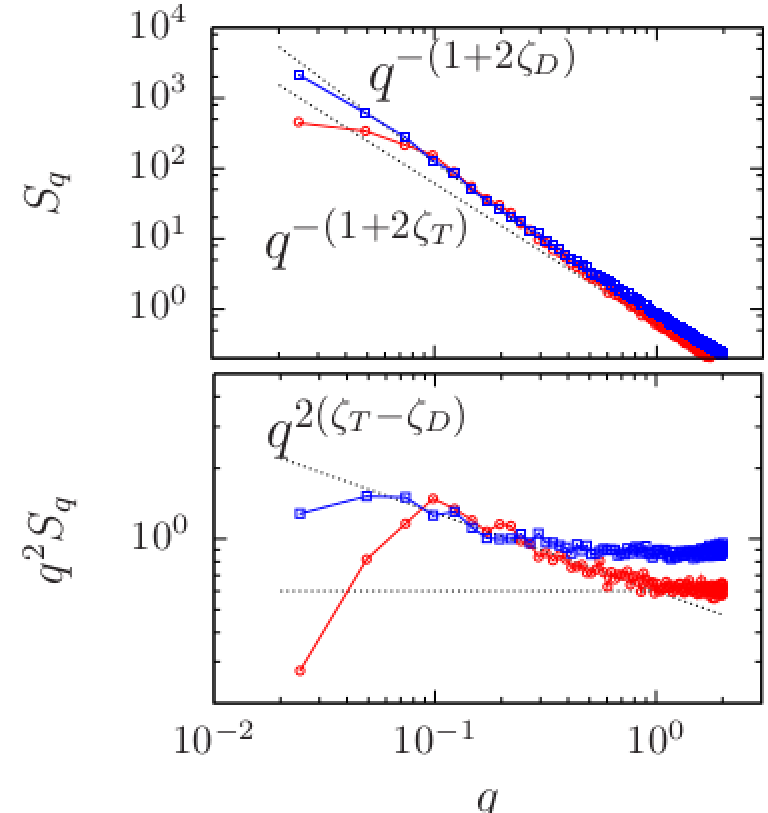
$$S(q) \sim \begin{cases} T q^{-(1+2\alpha_T)} & \text{for } q \gg q_T \\ q^{-(1+2\alpha)} & \text{for } q \ll q_T \end{cases}$$

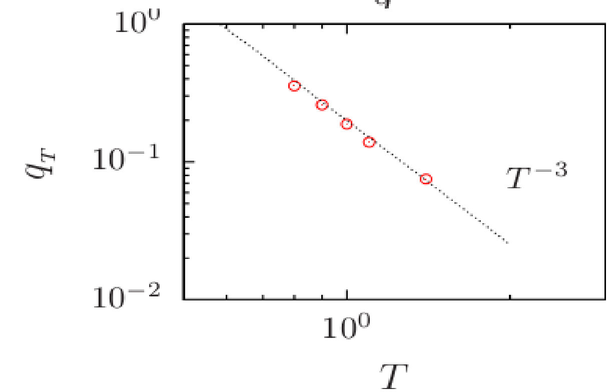
$$\alpha_T = \alpha_{\mathrm{EW}} = 1/2$$
$$\alpha = \alpha_{\mathrm{RM}} = 2/3$$

$L_T = 1/q_T$: **thermal length**

$$T q_T^{-(1+2\alpha_T)} \sim q_T^{-(1+2\alpha)}$$

$$L_T \sim T^{\frac{1}{2(\alpha-\alpha_T)}} \qquad \boxed{L_T \sim T^3}$$





Iguain et al,PRB, **80**, 094201, 2009

Thermal effects

BUT!!

$$L_T \sim T^{\frac{1}{2(\alpha_F - \alpha_T)}} \quad \text{with} \quad \alpha_F = 3/5$$

$$\boxed{L_T \sim T^5}$$

$\mathcal{H}_{\text{dis}}$ as a perturbation at very short length scales $r \ll L_T$

Nattermann, Shapir, Vilfan,PRB, **42**, 8577, 1990

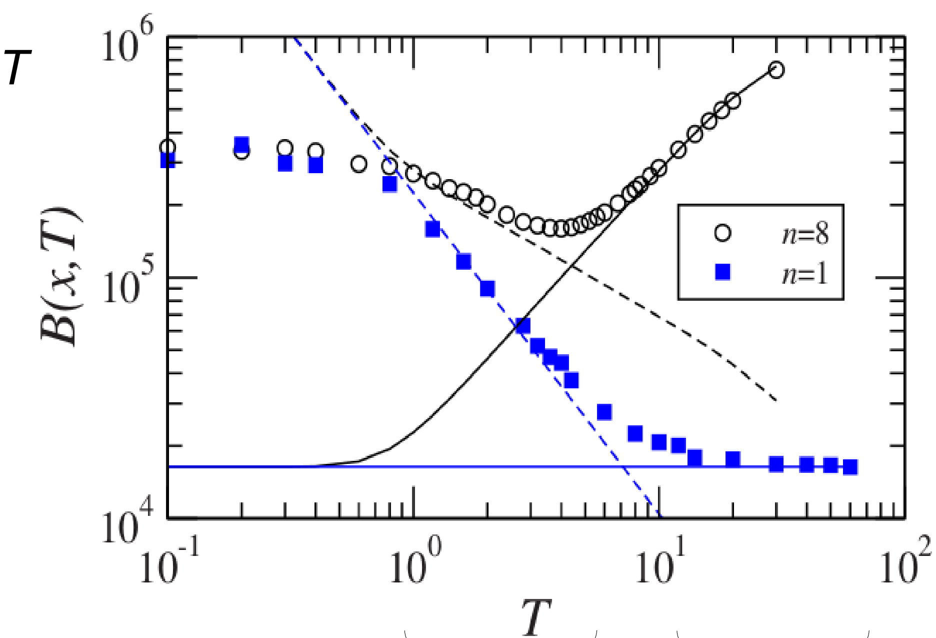
Variational Gaussian approach

Agoritsas, Lecomte, Giamarchi,PRB, 8**2**, 184207, 2010

directed polymer with transfer matrix at finite *T*

$$B(r,T) \sim \begin{cases} r^{2\alpha_T} & \text{for } r \ll L_T \\ T^{-4/3} r^{2\alpha} & \text{for } r \gg L_T \end{cases}$$

$$L_T \sim T^5/\nu \sim T^4$$



r is fixed   small L_T large L_T

Bustingorry, Le Doussal, Rosso,PRB, 8**2**, 140201, 2010

Thermal effects

BUT!!

$$L_T \sim T^{\frac{1}{2(\alpha_F - \alpha_T)}} \quad \text{with} \quad \alpha_F = 3/5$$

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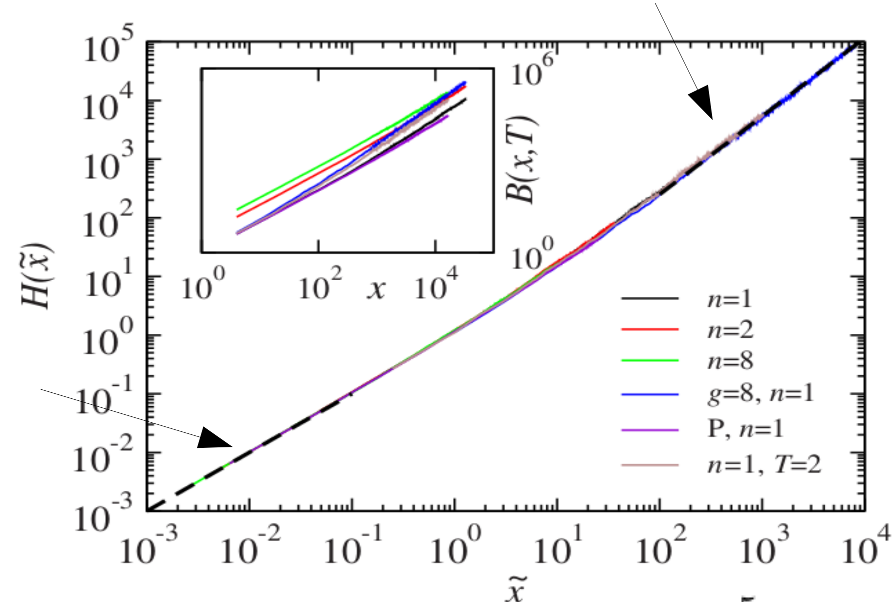
$$B(r,T) \sim \begin{cases} r^{2\alpha_T} & \text{for } r \ll L_T \\ T^{-4/3} r^{2\alpha} & \text{for } r \gg L_T \end{cases}$$

$$B(r,t) = \frac{T^6}{\nu^2} H\left[\frac{r}{(T^5/\nu)}\right] \qquad r^{2\alpha_T} \sim r$$

$$H(x) \sim \begin{cases} x & \text{for } x \ll L_T \\ x^{2\alpha} & \text{for } x \gg L_T \end{cases}$$

Bustingorry, Le Doussal, Rosso, PRB, 8**2**, 140201, 2010

$r^{2\alpha} \sim r^{4/3}$



$\sim r/(T^5/\nu)$

Quenched disorder

Thermal effects

$L_T \sim T^3$

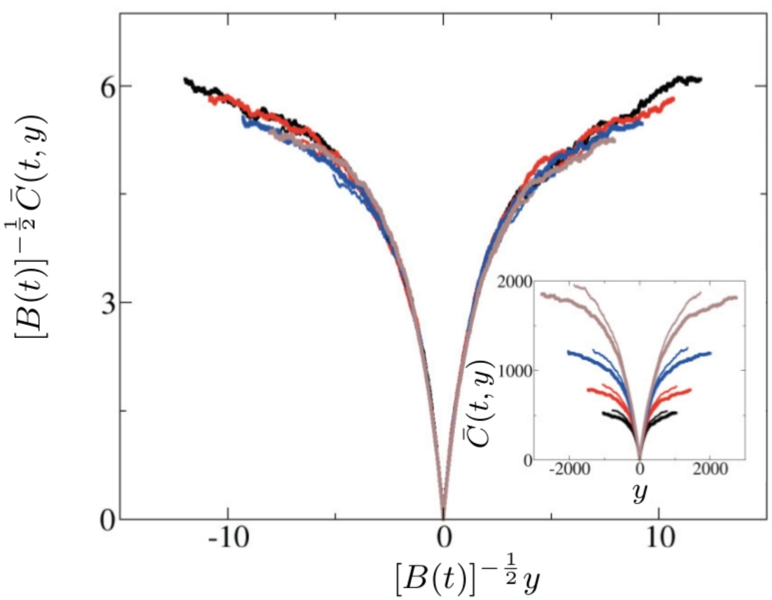
low-temperature regime

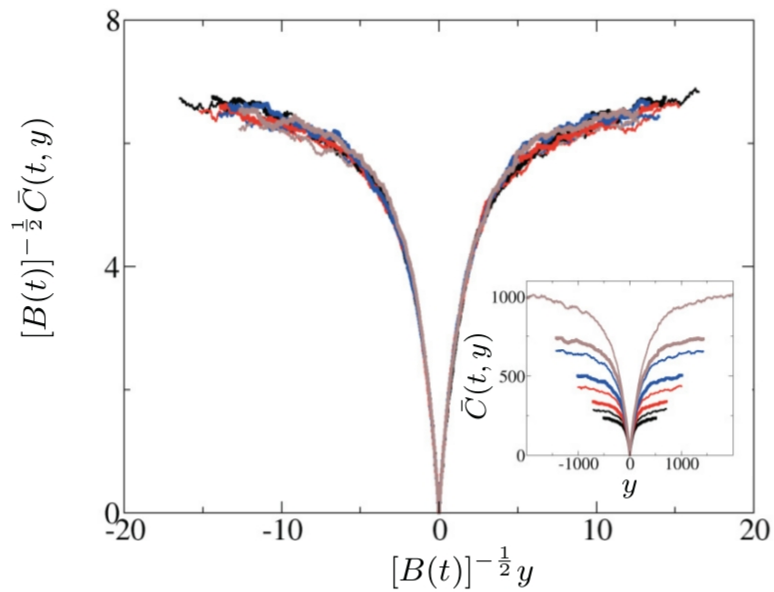
$L_T \sim T^5$

high-temperature regime

crossover??

free energy correlations





Elizabeth Agoritsas talk

Agoritsas, Lecomte, Giamarchi, PRB, **82**, 184207, 2010
Agoritsas, Lecomte, Giamarchi,PhysB, **407**, 1725, 2012
Agoritsas et al, PRE, **86**, 031144, 2012
Agoritsas, Lecomte, Giamarchi, PRB, **82**, 184207, 2013
Agoritsas, Lecomte, Giamarchi, PRB, **82**, 184207, 2013

Thermal effects

$L_T \sim T^3$

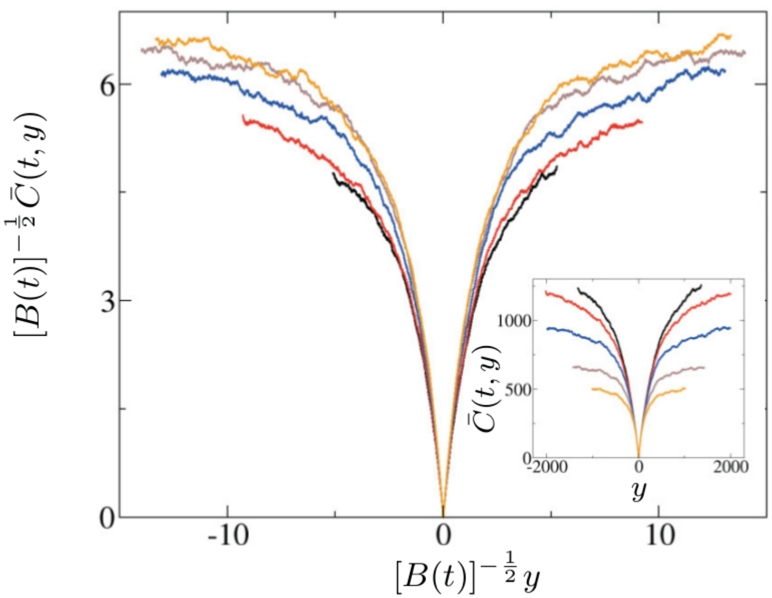
low-temperature regime

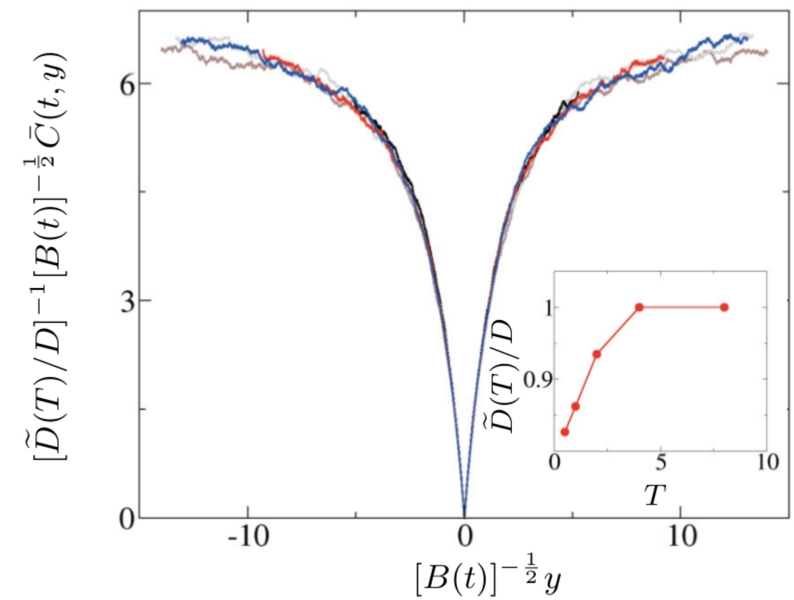
$L_T \sim T^5$

high-temperature regime

crossover??

free energy correlations

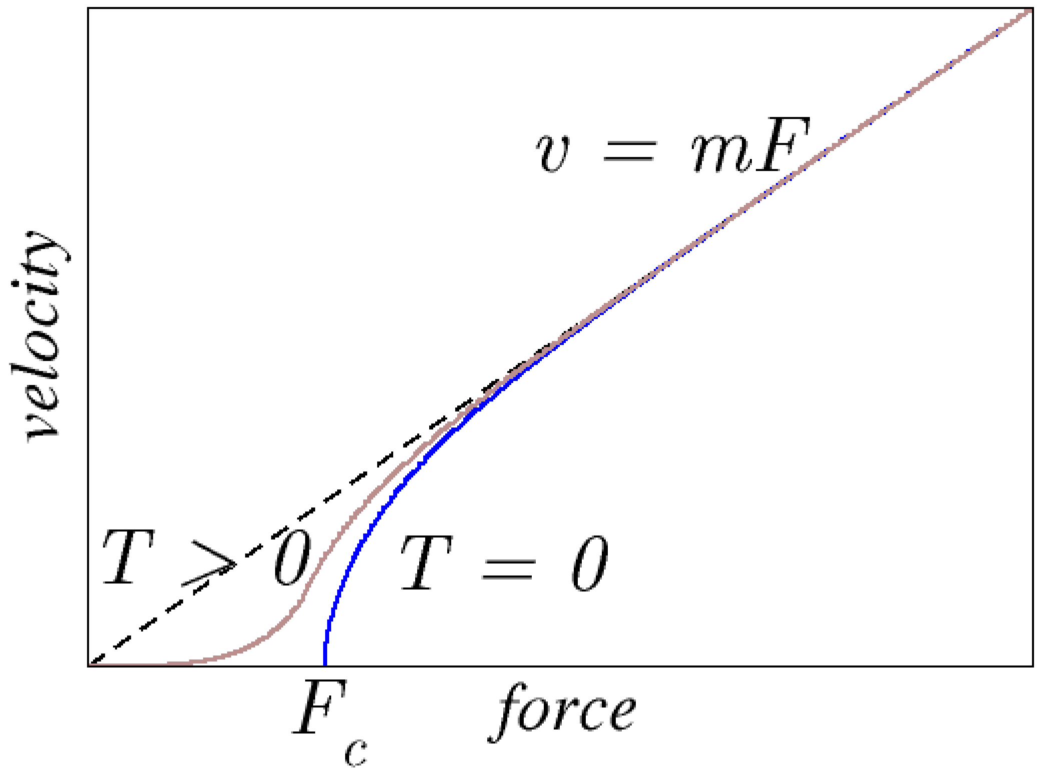




Agoritsas, Lecomte, Giamarchi, PRB, **82**, 184207, 2010
Agoritsas, Lecomte, Giamarchi,PhysB, **407**, 1725, 2012
Agoritsas et al, PRE, **86**, 031144, 2012
Agoritsas, Lecomte, Giamarchi, PRB, **82**, 184207, 2013
Agoritsas, Lecomte, Giamarchi, PRB, **82**, 184207, 2013

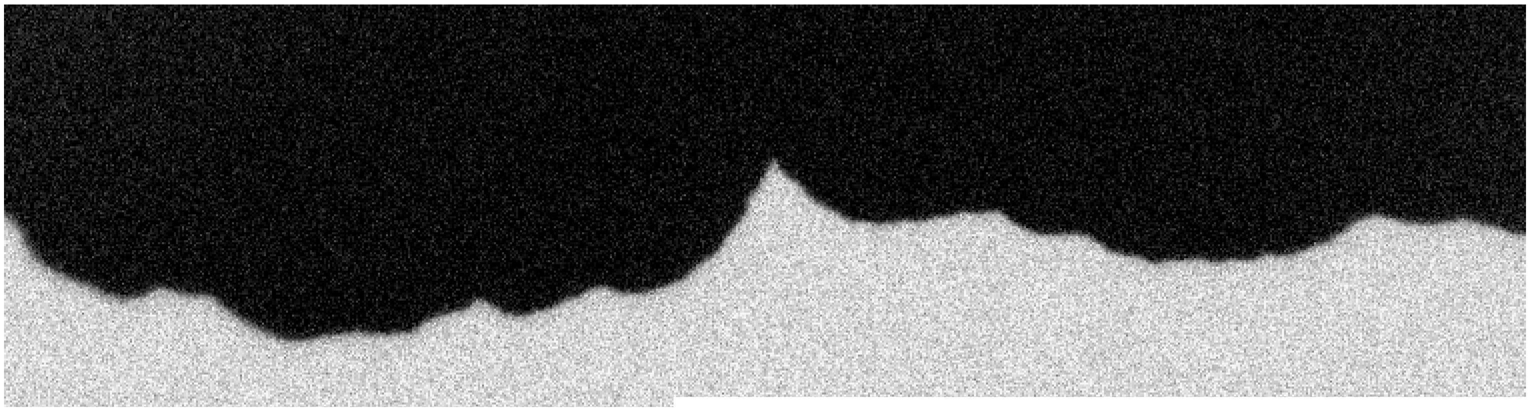
Elizabeth Agoritsas talk

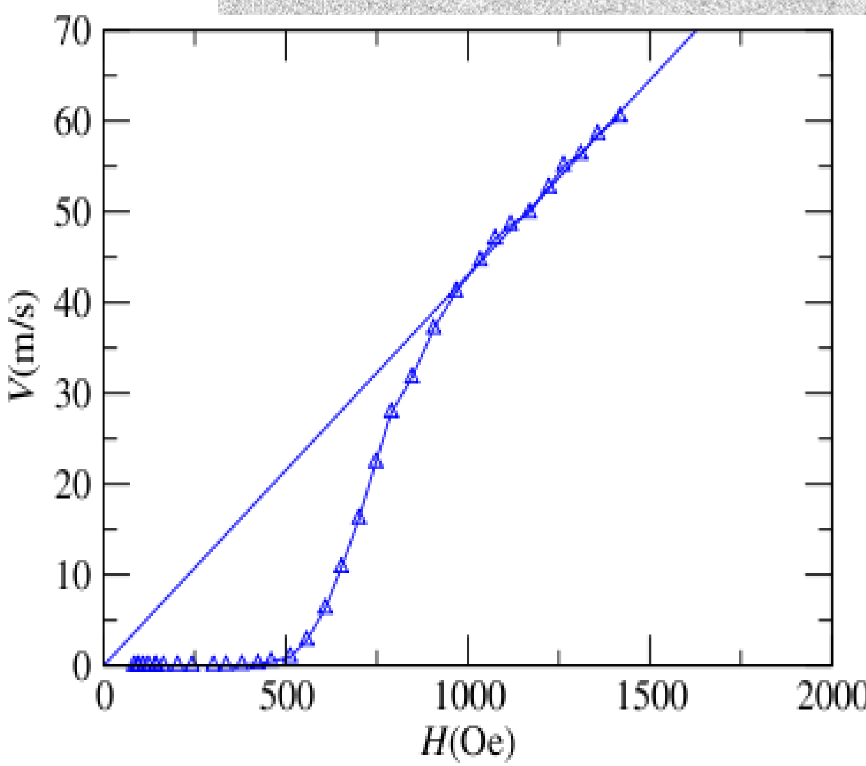
Depinning transition

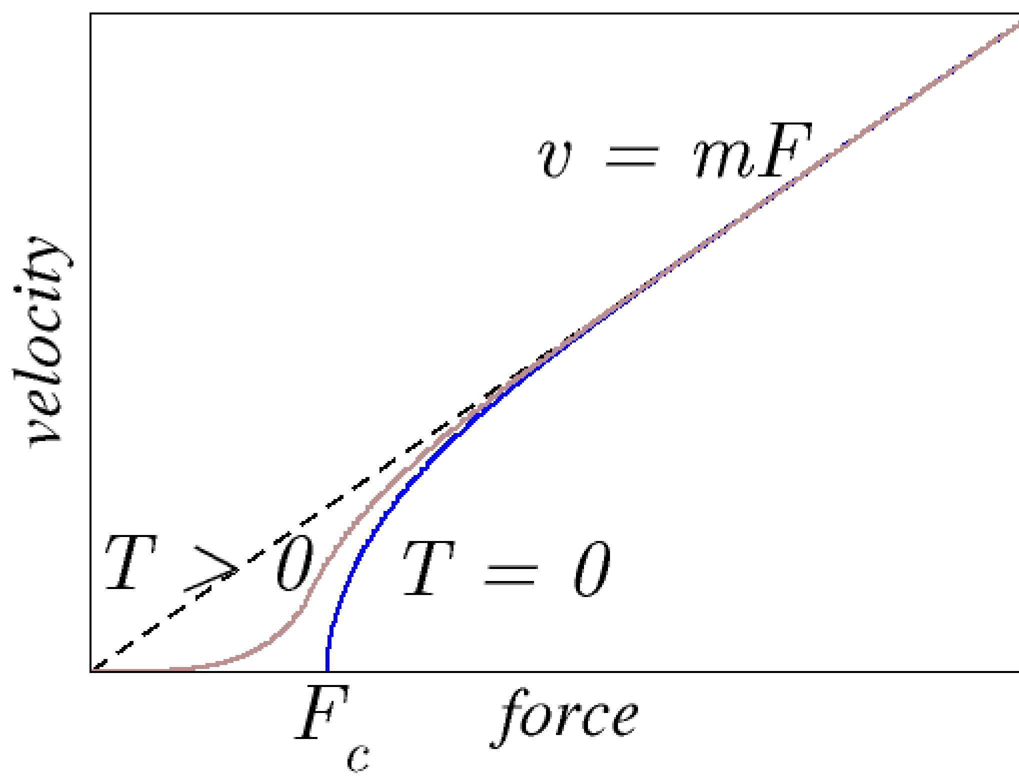


Quenched disorder

Depinning transition





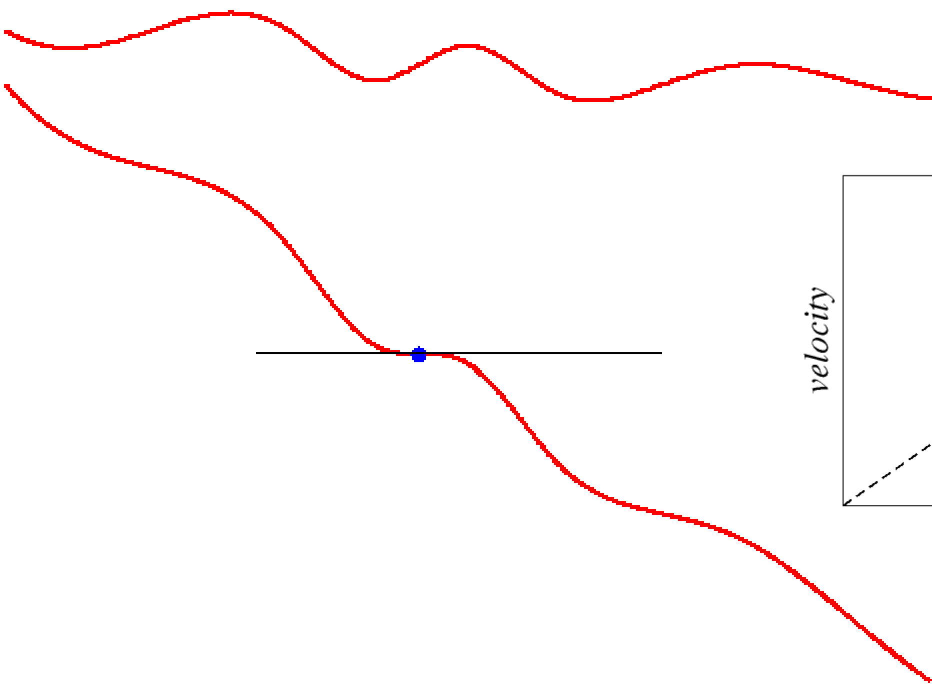


Metaxas et al,PRL, 8**2**, 140201, 2010

Depinning transition

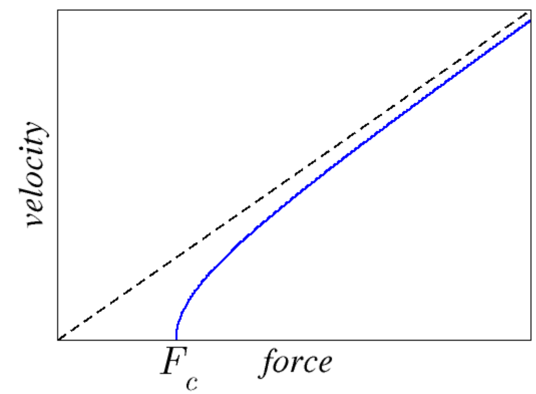
One dimension

particle in a random potential



$V(u)$

$V(u) - F_c\, u$



$$\mathcal{H} = V(u) - Fu$$

$$\gamma \partial_t u = -\partial_u \mathcal{H} = F - \partial_u V$$

$$\partial_t u > 0 \Rightarrow F > \partial_u V$$

$$F_c = \max_u \partial_u V$$

critical force

close to the critical position: $F \approx (F - F_c) + c\delta u^2$

$$\gamma \partial_t u = \delta f + c\delta u^2$$

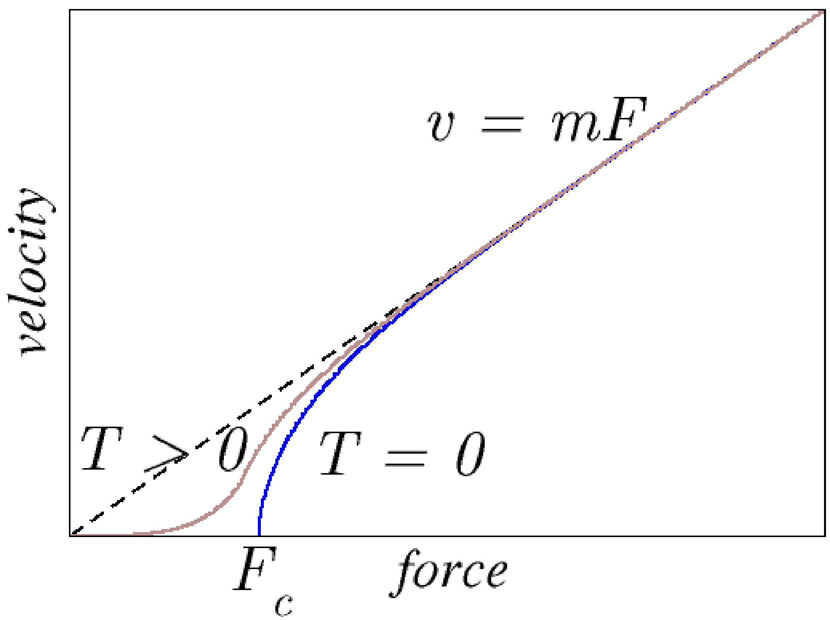
$$\tau = \gamma \int \frac{du}{\delta f + c\delta u^2} = \frac{\gamma}{2c\sqrt{\delta f}} \int \frac{d\tilde{u}}{\sqrt{\tilde{u}}(1+\tilde{u}^2)} \sim \frac{1}{\sqrt{\delta f}}$$

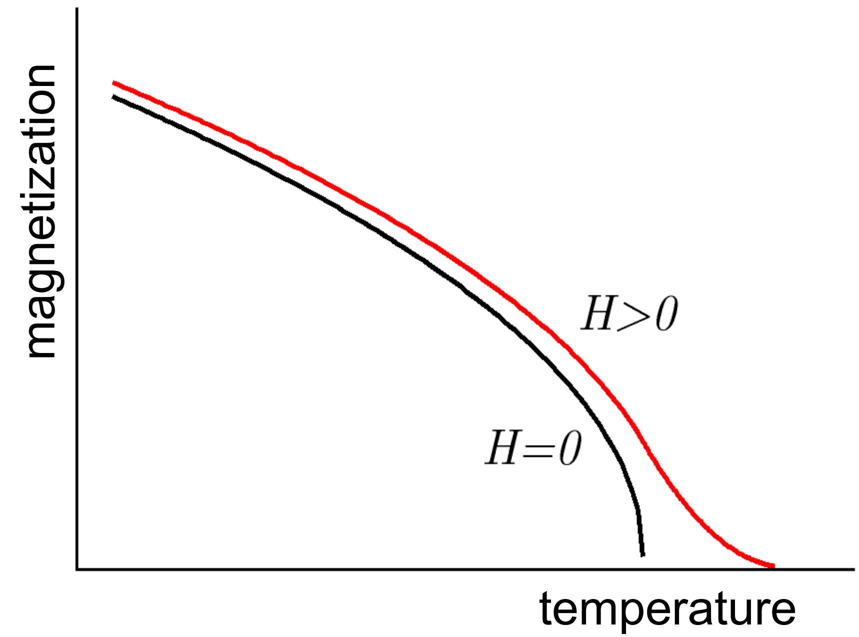
typical time spent close to the critical position

$$V \sim \tau^{-1} \sim \delta f^{1/2}$$

velocity – force characterisic

Depinning transition





$V \sim (F - F_c)^{\beta}$	order parameter	$M \sim (T - T_c)^{\beta}$
$\xi \sim (F - F_c)^{-\nu}$	correlation length	$\xi \sim (T - T_c)^{-\nu}$
$V \sim T^{\psi}$	field rounding	$M \sim h^{1/\delta}$

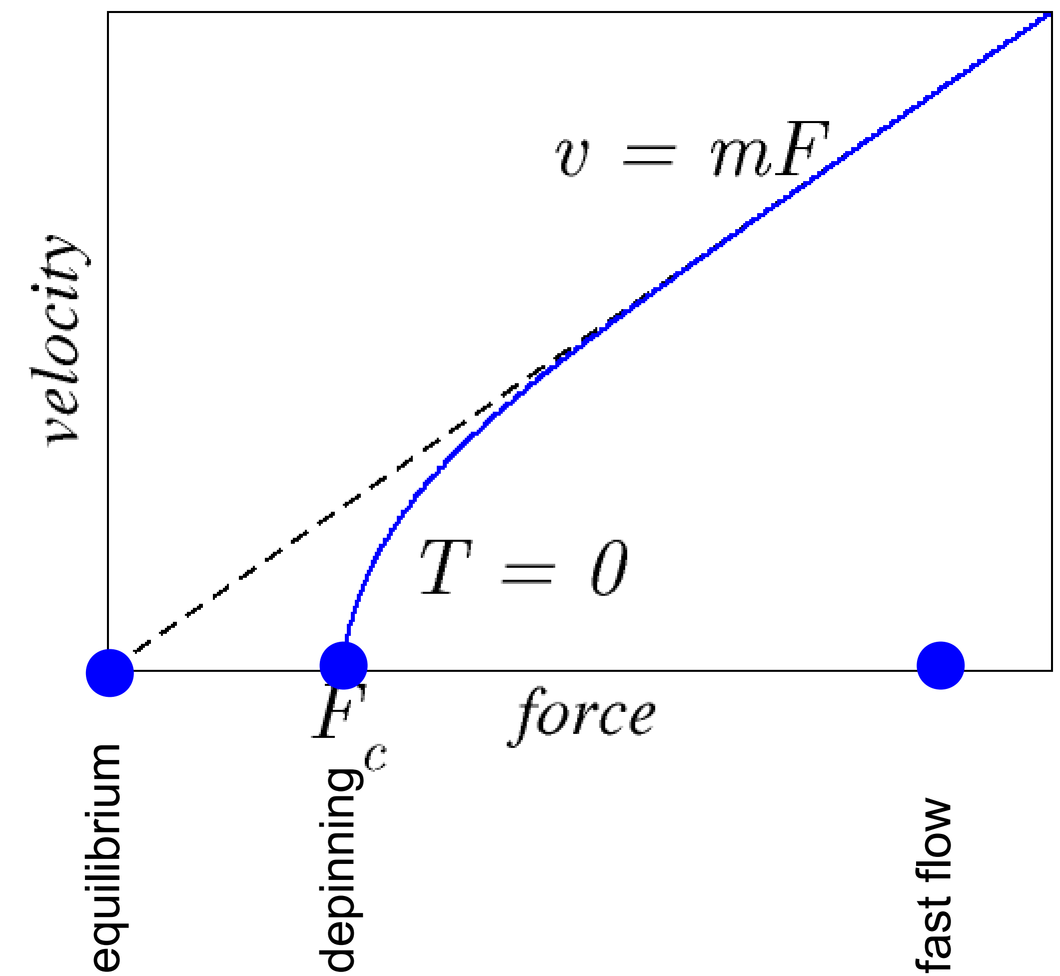
Depinning transition   Geometrical regimes

equilibrium $\zeta_{eq} = 2/3$

depinning $\zeta_{dep} = 1.25$

fast flow $\zeta_{FF} = 1/2$

three geometrical “fixed points” describing all length scales (harmonic, RB)

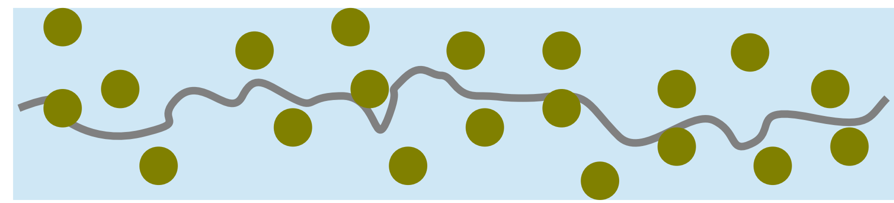


Depinning transition

Geometrical regimes

$$\mathcal{H}[u] = \int_L dz \left[\frac{c}{2} \left(\frac{\partial u}{\partial z} \right)^2 + V(u, x) \right]$$

competition between elasticity and disorder



$$\boxed{\zeta_{eq} = 2/3}$$

equilibrium roughness exponent
(zero force and zero temperature)

$$\zeta_{eq} > \zeta_{\mathrm{random-walk}} = 1/2$$

$$B(r) = \overline{\left\langle \left[u(z, r) - u(z) \right]^2 \right\rangle}$$

$B(r) \sim r^{2\zeta}$ with ζ the **roughness exponent**

power-law behavior, signature of self-affine properties

the interface adjust to the disorder
environment and roughens

Depinning transition

Geometrical regimes

fast flow

$$\partial_t u(z,t) = \nu \partial_z^2 u(z,t) + \xi(u,z) = \nu \partial_z^2 u(z,t) + \xi(vt,z)$$

$$\begin{aligned} \overline{\tilde{\xi}(t,z)} &= \xi(vt,z) \\ \overline{\tilde{\xi}(t,z)\tilde{\xi}(t',z')} &= \frac{D}{v}\delta(t-t')\delta(z-z') \end{aligned}$$

disorder becomes an effective “thermal” noise of intensity $\frac{D}{v}$

$$\zeta_{FF} = \zeta_{\text{thermal}} = \zeta_{\text{random-walk}}$$

$$\boxed{\zeta_{FF} = 1/2}$$

Depinning transition

Geometrical regimes

fast flow

$$\partial_t u(z,t) = \nu \partial_z^2 u(z,t) + \xi(u,z) = \nu \partial_z^2 u(z,t) + \xi(vt,z)$$

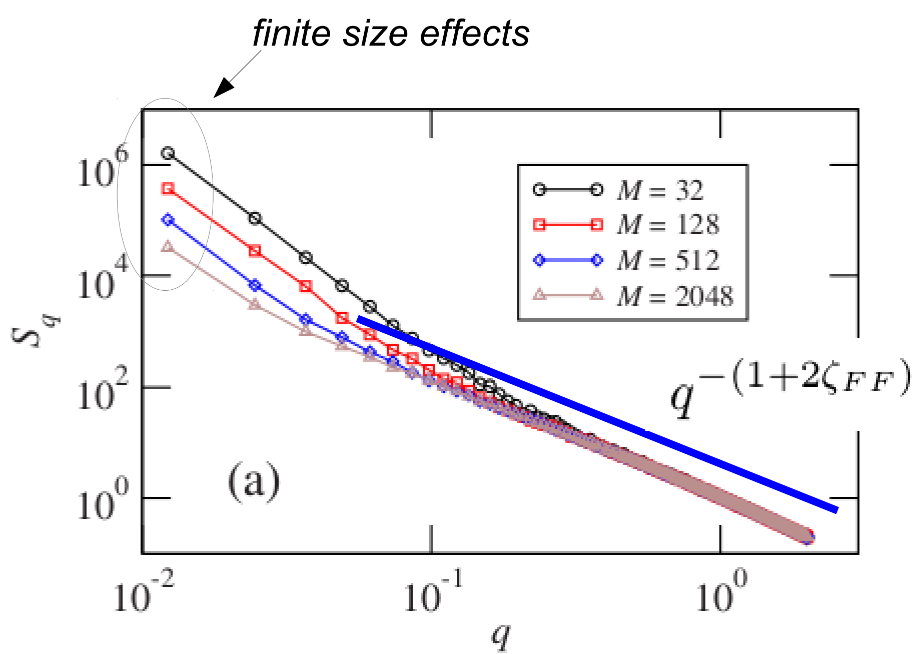
$$\begin{aligned} \overline{\tilde{\xi}(t,z)} &= \xi(vt,z) \\ \overline{\tilde{\xi}(t,z)\tilde{\xi}(t',z')} &= \frac{D}{v}\delta(t-t')\delta(z-z') \end{aligned}$$

$$\boxed{\zeta_{FF} = 1/2}$$

$$S(q,t) = \langle u(q,t)u(-q,t)\rangle$$

$$B(r,t) = \int \frac{dq}{\pi}\,[1-\cos(qr)]\,S(q)$$

same information if $\zeta < 1$



Depinning transition

Geometrical regimes

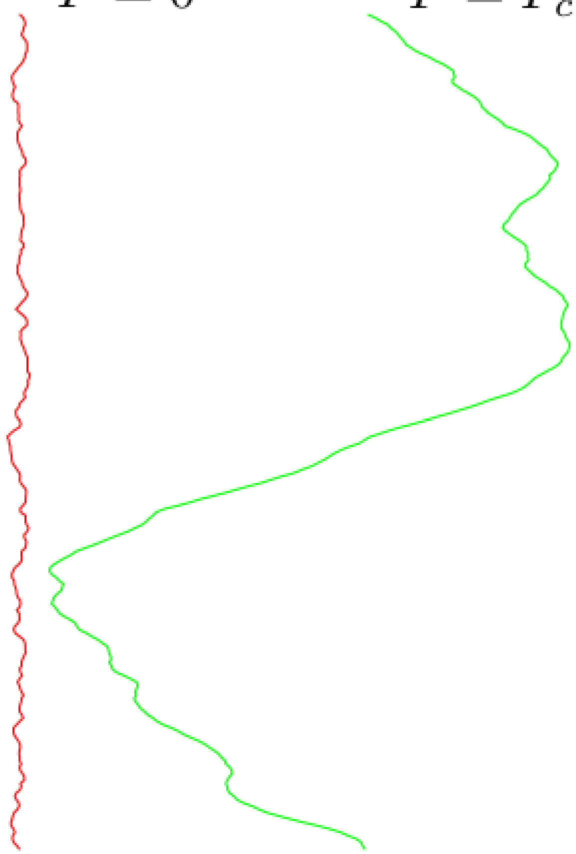
depinning configuration

the interface becomes rougher: it is ready to move but stand still

"infinite" avalanche ready to move

$$\xi \sim (F - F_c)^{-\nu}$$

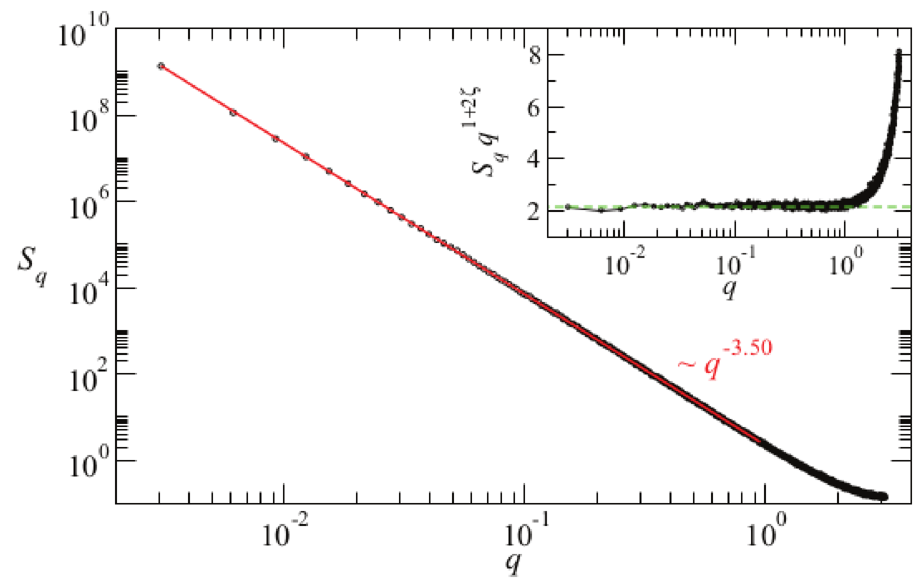
$F = 0$ $\quad$ $F = F_c$



$$S(q) \sim q^{-(1+2\zeta_{dep})}$$

$$\boxed{\zeta_{dep} = 1.25}$$

$B(r) \sim r$ (anomalous scaling)



Rosso, Krauth, PRE, 2001

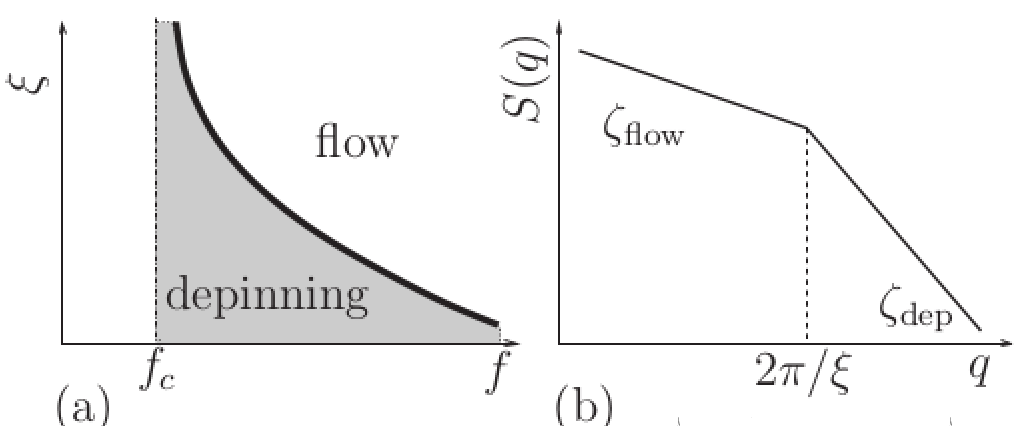
Ferrero, Bustingorry, Kolton, PRE 2013

Depinning transition

Geometrical regimes

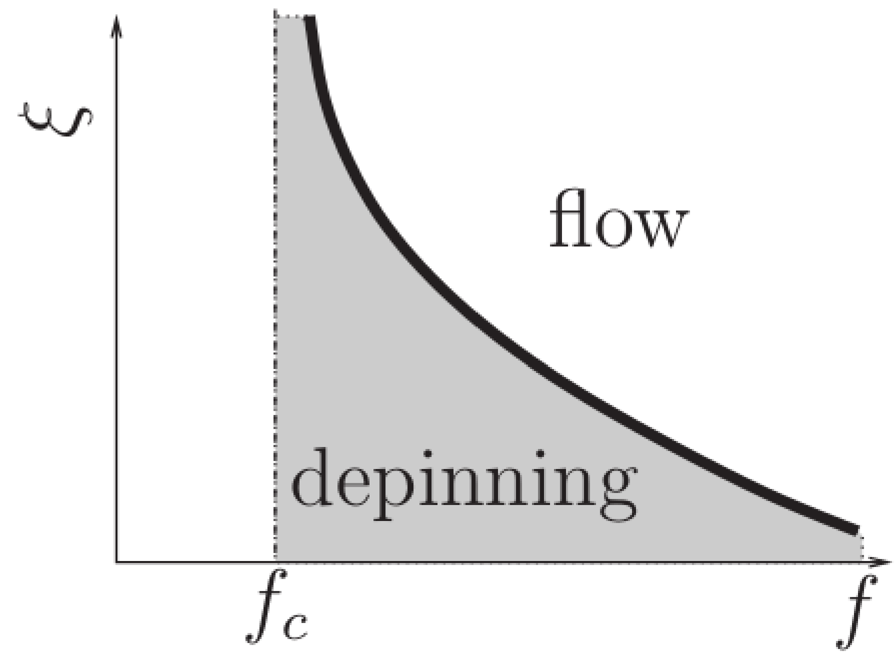
typical size of an “avalanche”
which is critically pinned and detach

$$\xi \sim (F - F_c)^{-\nu}$$



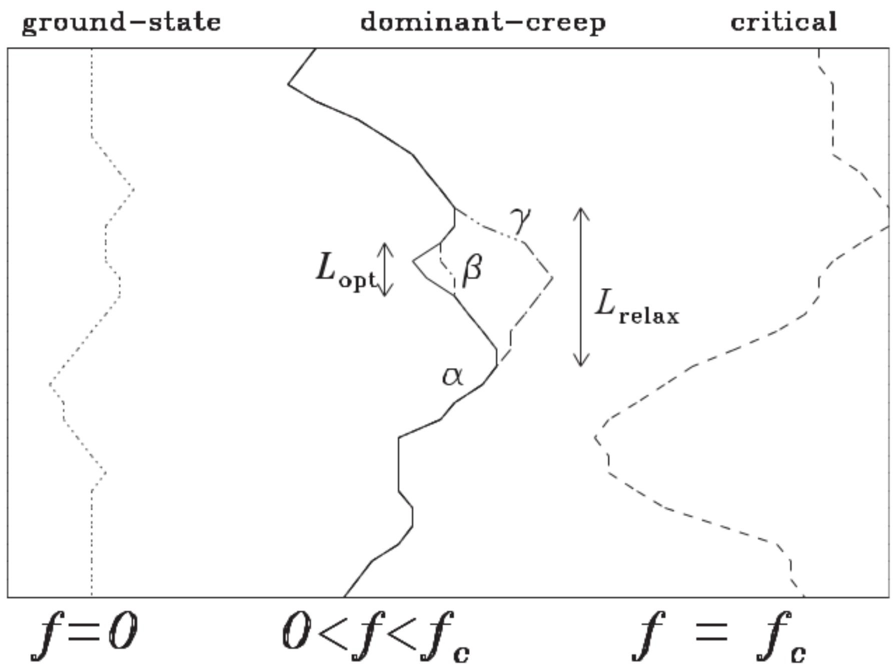
large length scale

small length scale



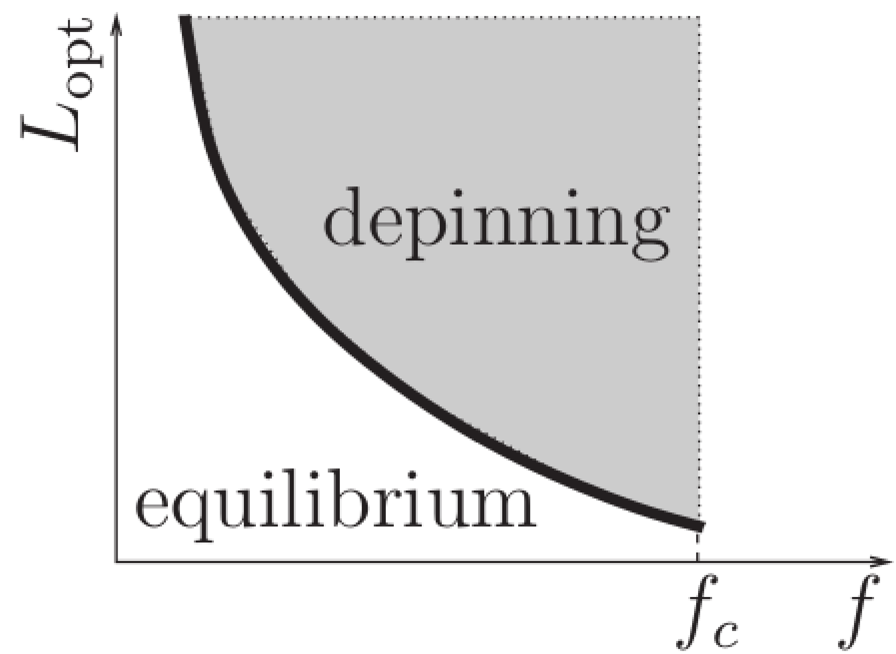
Depinning transition

Geometrical regimes



$$L_{opt} \sim F^{-\nu_{eq}} \qquad \nu_{eq} = 3/4$$

SCALING GAME!!!



sequence of metastable states

L_{opt} optimal size of the interface, necessary to excite to go to the next metastable state (steady state property)

L_{relax} relaxed size of the next metastable state (transient dynamics)

Kolton et al, PRL, 2006; PRB 2009

Depinning transition

Geometrical regimes

energy scale

$$\mathcal{H} \sim \int dz (\partial_z u)^2 \Rightarrow U \sim \ell^{2\zeta+d-2} \sim \ell^{\theta}$$

energy exponent $\theta = 2\zeta_{eq} + d - 2$

energy gained by the force

$$F\ell^d w = F\ell^d r_f \left(\frac{\ell}{L_c}\right)^{\zeta} = F L_c^d r_f \left(\frac{\ell}{L_c}\right)^{d+\zeta}$$

$$U_T = U_c \left(\frac{\ell}{L_c}\right)^{\theta} - F L_c^d r_f \left(\frac{\ell}{L_c}\right)^{d+\zeta}$$

maximum berrier

$$\left.\frac{\partial U_T}{\partial \ell}\right|_{L_{opt}} = \frac{U_c \theta}{L_c}\left(\frac{L_{opt}}{L_c}\right)^{\theta-1} - F L_c^{d-1} r_f (d+\zeta)\left(\frac{L_{opt}}{L_c}\right)^{d+\zeta-1} = 0$$

$$\left(\frac{L_{opt}}{L_c}\right)^{d+\zeta-1-\theta+1} = A\left(\frac{F_c}{F}\right) \qquad F_c = \frac{c r_f}{L_c^2}$$

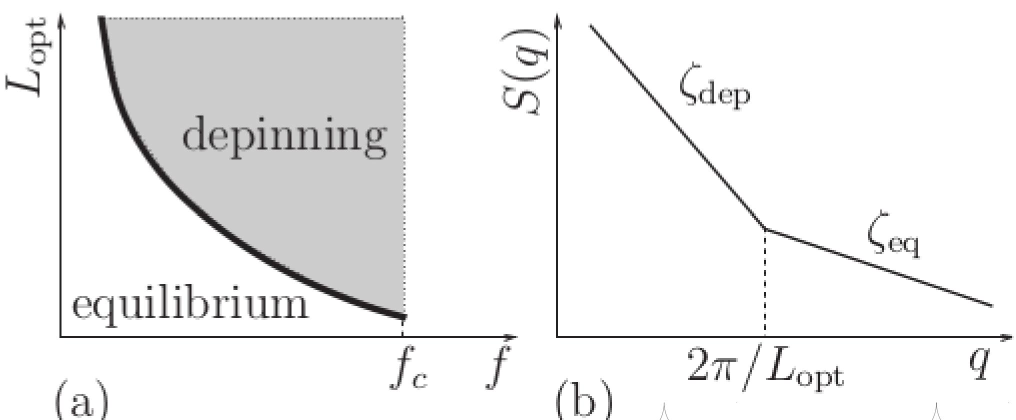
$$d + \zeta - 1 - \theta + 1 = 2 - \zeta$$

$$L_{opt} = L_c \left(\frac{F}{F_c}\right)^{-1/(2-\zeta_{eq})} = L_c \left(\frac{F}{F_c}\right)^{-\nu_{eq}}$$

Depinning transition   Geometrical regimes

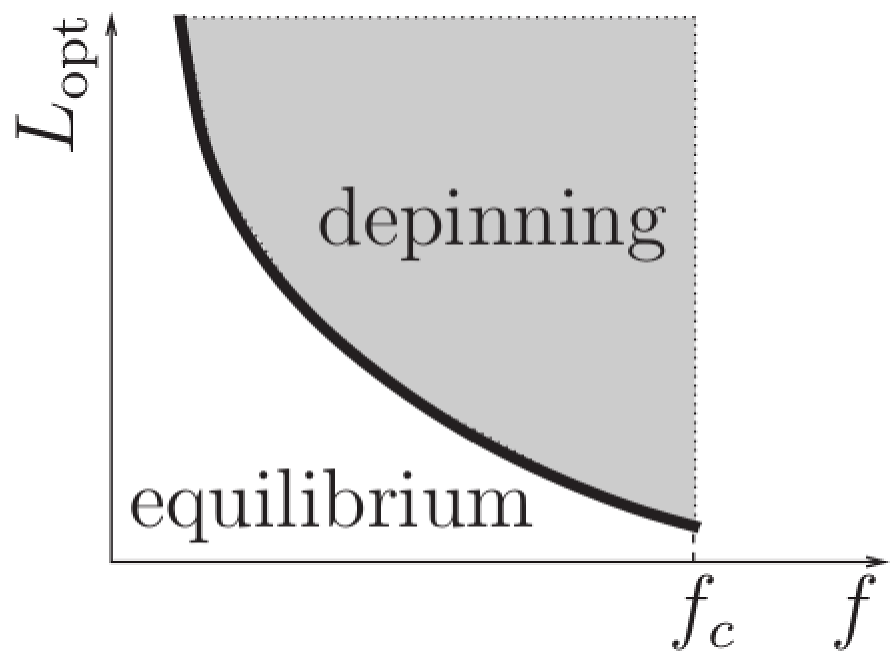
$$L_{opt} \sim F^{-\nu_{eq}} \qquad \nu_{eq} = 3/4$$

SCALING GAME!!!



large length scale

small length scale

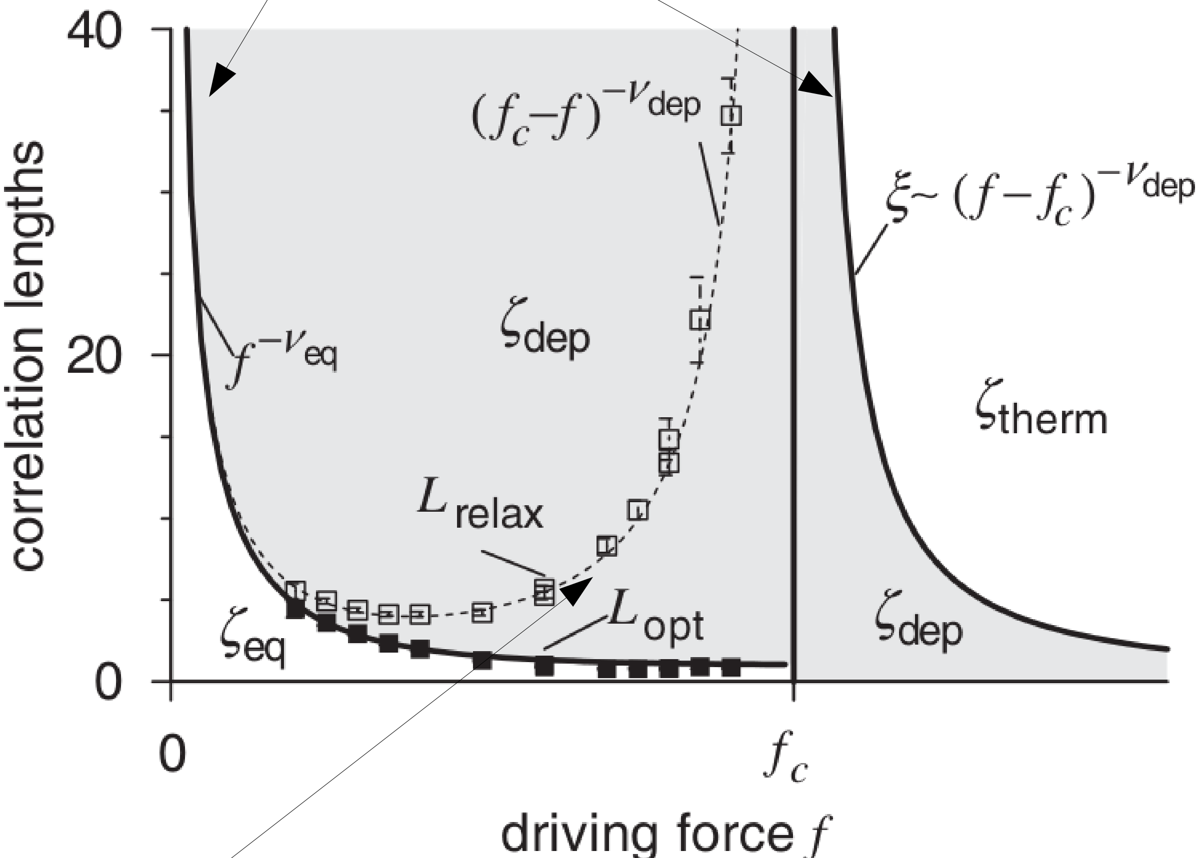


Kolton et al, PRL, 2006; PRB 2009

Depinning transition

Geometrical regimes

divergent steady state length scales

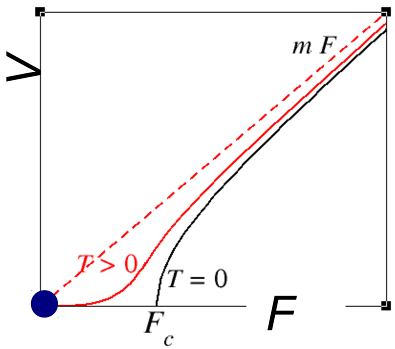


dynamic length

not as in standard critical phenomena

Kolton et al, PRL, 2006; PRB 2009

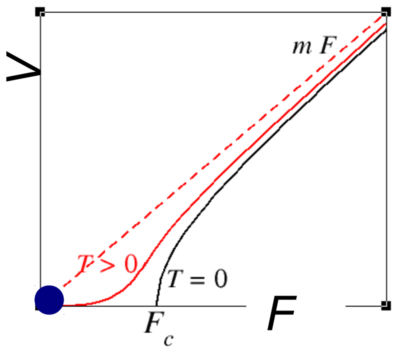
Quenched disorder



Depinning transition

F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

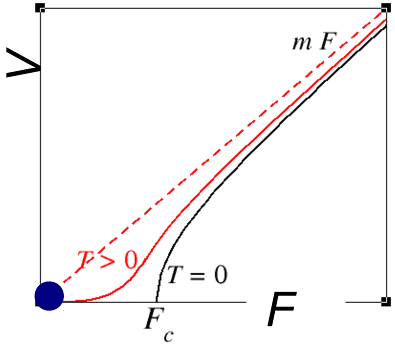
Quenched disorder



F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

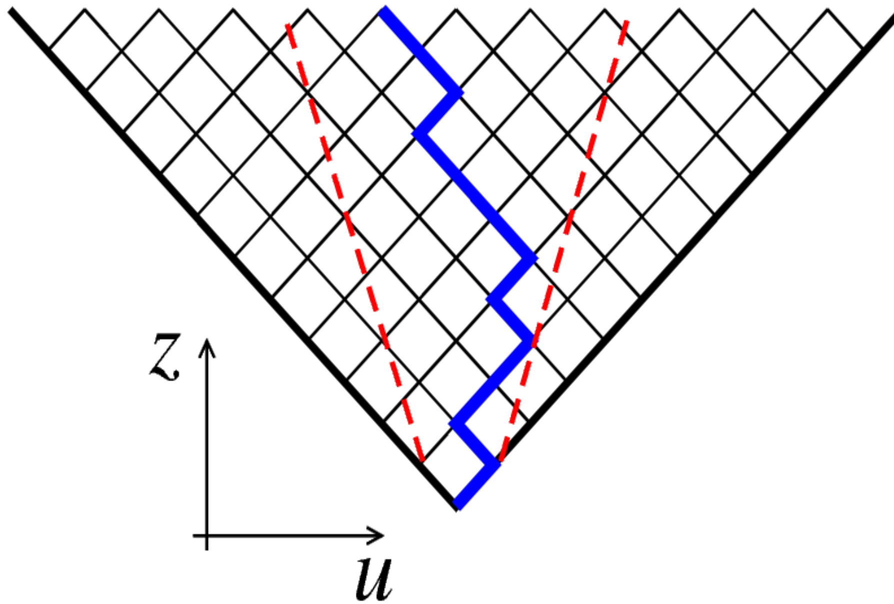
Depinning transition

Quenched disorder



F	T	Δ
0	**0**	**0**
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

Depinning transition



$$w^2 = \langle [u(z) - \langle u \rangle]^2 \rangle = \frac{T}{c} z$$

Random walk

$$R^2 = 2Dt$$

$$w \sim z^{\zeta}$$

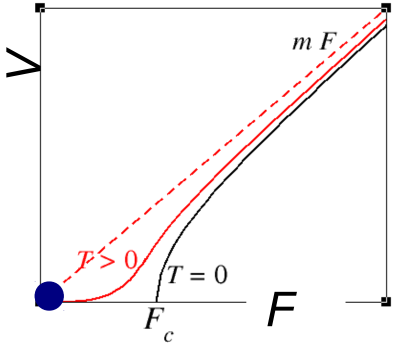
$$\zeta$$

$$\zeta_{th} = 1/2$$

roughness exponent

$$S_q = \langle u_q u_{-q} \rangle \frac{T}{c} = \frac{1}{q^2} \sim q^{-(1+2\zeta)}$$

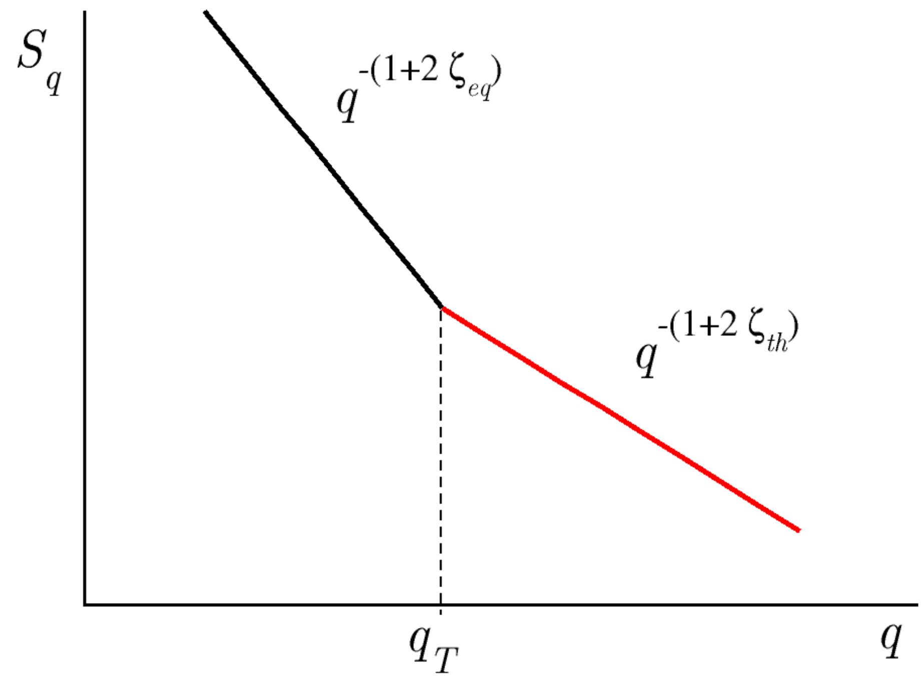
Quenched disorder



F	T	Δ
0	**0**	**0**
0	**>0**	**0**
0	**0**	**>0**
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

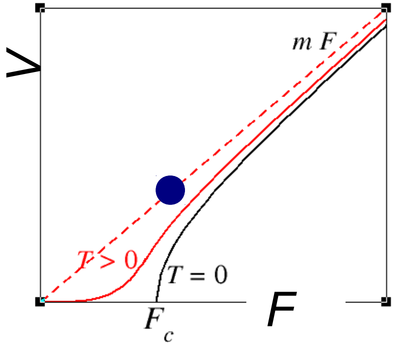
Depinning transition

$$w \sim r^{\zeta_{eq}},\ \zeta_{eq} = 2/3,\ \zeta_{eq} > \zeta_{th}$$



$$L_T \sim \frac{1}{q_T} \sim T^5$$

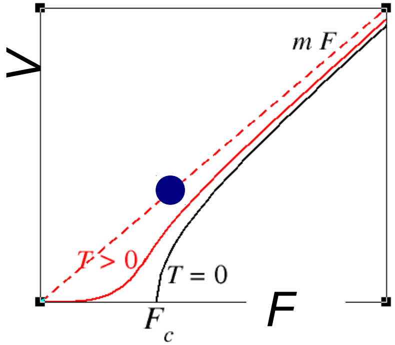
Quenched disorder



F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

Depinning transition

Quenched disorder

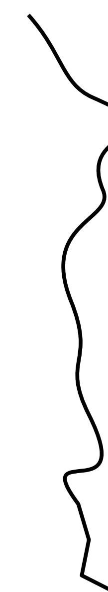


F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

Depinning transition

T = 0

T > 0

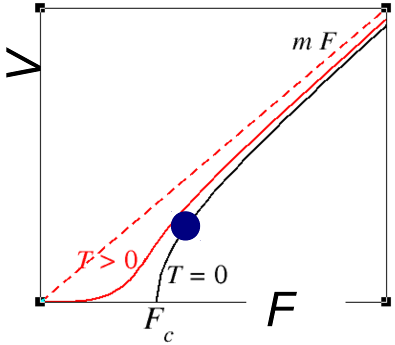


$\zeta = 0$

$\zeta = \zeta_{th}$

V= m F

Quenched disorder



F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

Depinning transition

DEPINNING!!

fast-flow

$$F \gg F_c \Rightarrow V = mF; \quad \zeta = \zeta_{th}$$

depinning

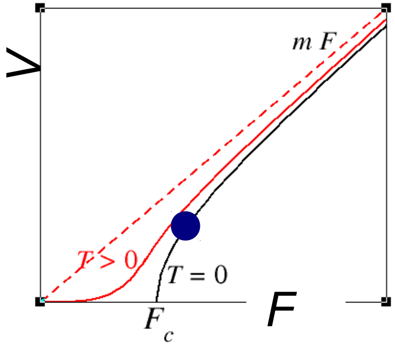
$$F < F_c \Rightarrow V = 0; \quad \zeta = \zeta_{eq}$$

$$F = F_c \Rightarrow V = 0; \quad \zeta = \zeta_{dep}$$

$$F \gtrsim F_c \Rightarrow \begin{cases} V \sim (F - F_c)^{\beta}; & \beta = 1/4 \\ \xi \sim (F - F_c)^{-\nu}; & \nu = 4/3 \end{cases}$$

β :depinning exponent
ν :correlation length exponent

Quenched disorder



F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

Depinning transition

DEPINNING!!

$$F \leq F_c \quad \Rightarrow \quad V = 0$$

$$F \gtrsim F_c \quad \Rightarrow \quad \begin{cases} V \sim (F - F_c)^{\beta} \\ \xi \sim (F - F_c)^{-\nu} \end{cases}$$

$\beta = 0.25$:depinning exponent

$\nu = 4/3$:correlation length exponent

$$\beta = 0.245 \pm 0.006$$

$$\nu = 1.333 \pm 0.007$$

numerical studies of relaxation properties in extremely large elastic line systems

$$V \sim (F - F_c)^{\beta} \sim \xi^{-\beta/\nu} \sim t^{-\beta/\nu z}$$

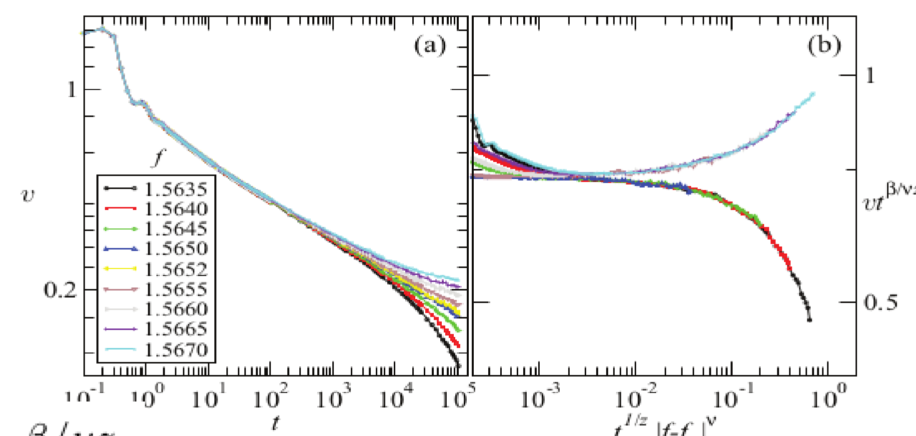
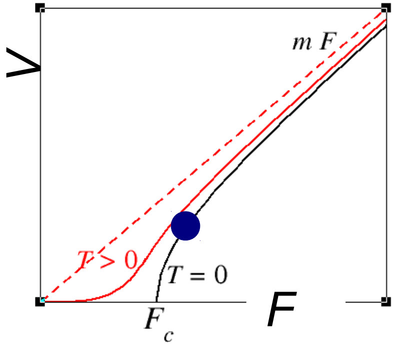


FIG. 13. (Color online) String velocity $v(t)$ as a function of time for the RB case with uniformly distributed disorder for which $f_c = 1.5652$ using $\delta t = 0.1$. The system size is $L = 4\,194\,304$. In (a) we present the raw data, and in (b) $v(t,f)$ has been rescaled to $vt^{\beta/\nu z}$ and t to $t^{1/z}|f - f_c|^{\nu}$.

Ferrero, Bustingorry, Kolton, PRE 2013

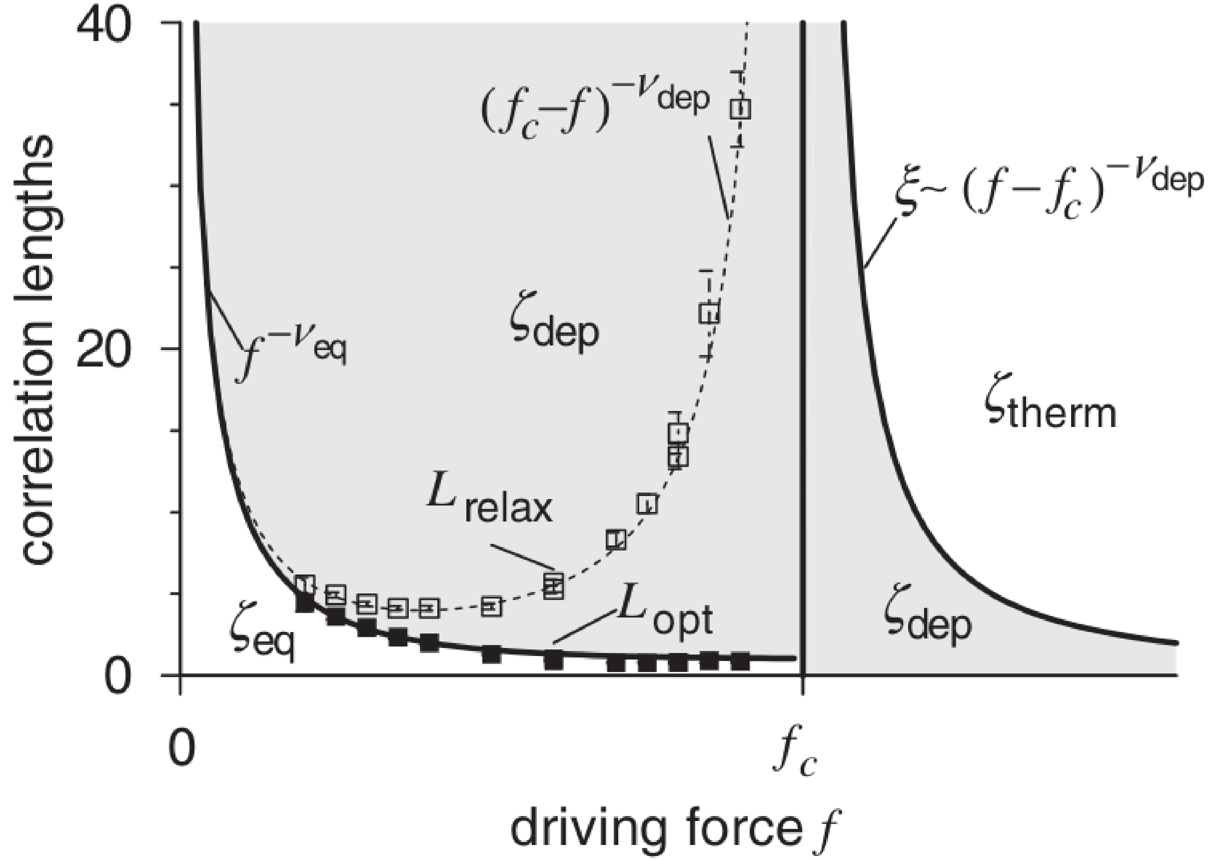
Quenched disorder



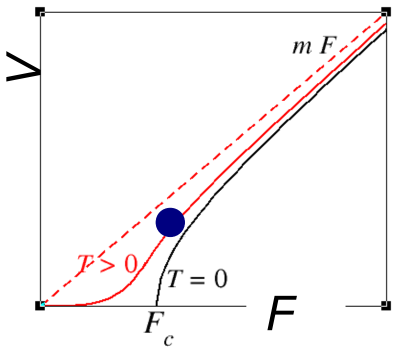
F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

Depinning transition

DEPINNING!!



Quenched disorder



F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

Depinning transition

Thermal rounding

fast – flow (sliding)

$$F \gg F_c \Rightarrow V = mF$$

creep

$$F \ll F_c \Rightarrow V = V_0 e^{-\frac{U_c}{k_B T}\left(\frac{F}{F_c}\right)^{-\mu}}$$

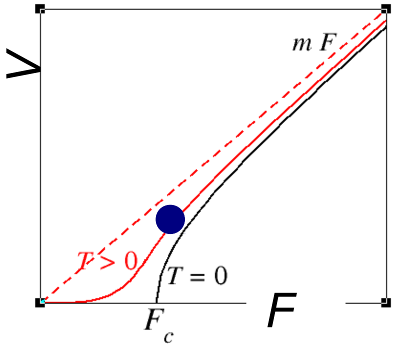
thermal rounding

$$F = F_c \Rightarrow V \sim T^{\psi}$$

μ :creep exponent

ψ :thermal rounding exponent

Quenched disorder



F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

Depinning transition

Thermal rounding

creep

$$F \ll F_c \Rightarrow V = V_0 e^{-\frac{U_c}{k_B T}\left(\frac{F}{F_c}\right)^{-\mu}}$$

the movement is achieved by overcoming the barriers associated to the optimal length

$$\mu = \frac{2\zeta_{eq} + d - 2}{2 - \zeta_{eq}}$$

$$U \sim L_{opt}^{\theta} \sim F^{-\theta\nu}$$

$$U(F) = U_c \left(\frac{F}{F_c}\right)^{-\mu}$$

velocity is given by Arrhenius activation over this characteristic energy scale

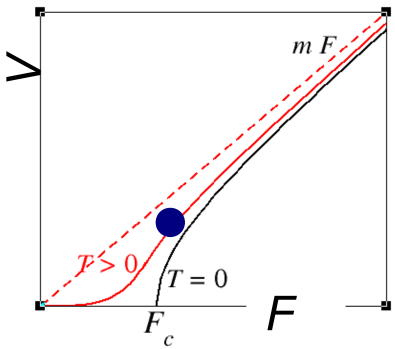
$$V = V_0 \exp\left(-\frac{U}{k_B T}\right) = V_0 \exp\left[-\frac{U_c}{k_B T}\left(\frac{F}{F_c}\right)^{-\mu}\right]$$

μ **:creep exponent**

$(d = 1)$ $\mu = 1/4$

Nattermann, PRL 1990
Chauve, Giamarchi, Le Doussal, PRB 2002

Quenched disorder



F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

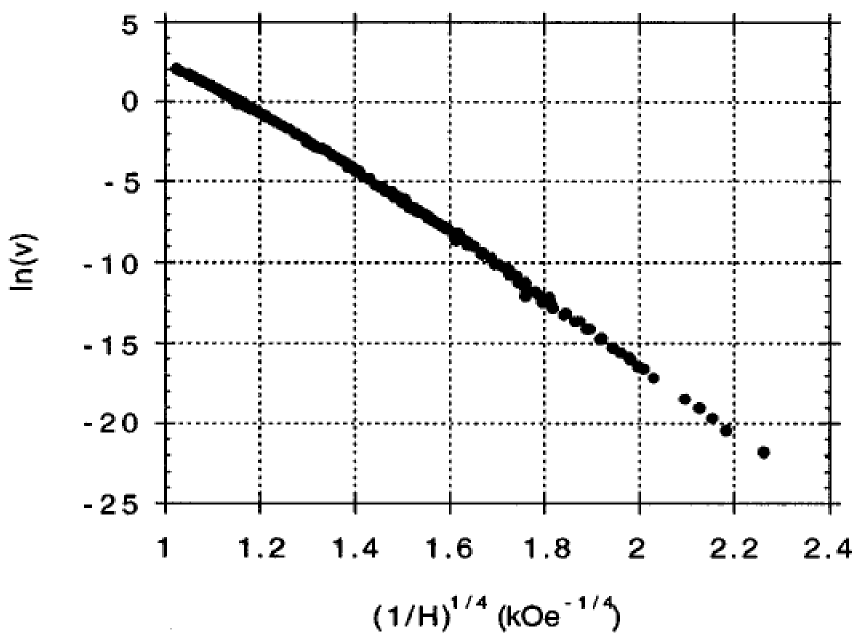
Depinning transition

Thermal rounding

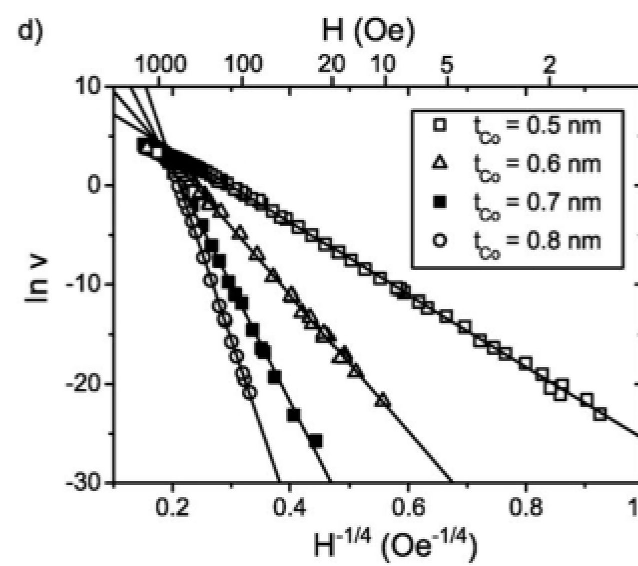
$$\mu = 1/4$$

robust exponent!

$$V = V_0 \exp\left(-\frac{U}{k_B T}\right) = V_0 \exp\left[-\frac{U_c}{k_B T}\left(\frac{F}{F_c}\right)^{-\mu}\right]$$

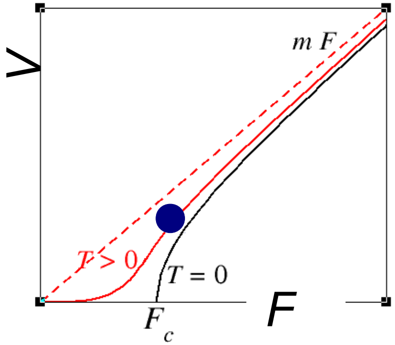


Pt/Co/Pt
ferromagnetic ultrthin film (0.5nm)
LPS, Orsay
Lemerle et al, PRL, 1998



Metaxas et al, PRL, 2007

Quenched disorder



F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

Depinning transition

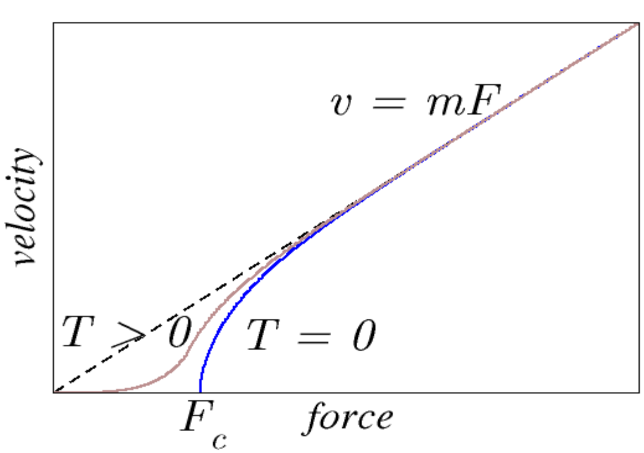
Thermal rounding

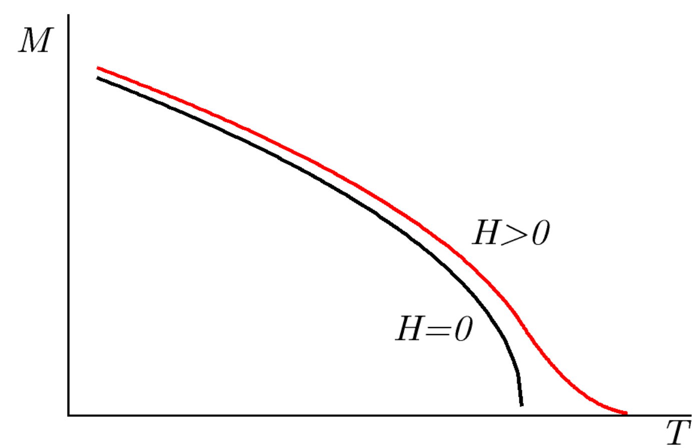
finite temperature depinning: **Thermal rounding**

from standard critical phenomena

$$F = F_c \Rightarrow V \sim T^{\psi}$$

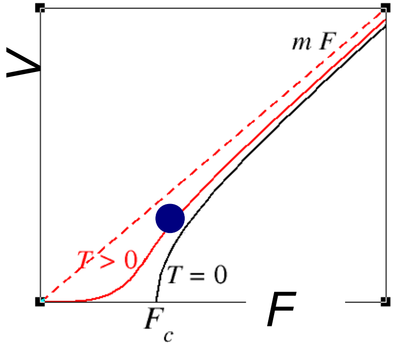
ψ **:thermal rounding exponent**





$V \sim T^{\psi}$ field rounding $M \sim h^{1/\delta}$

Quenched disorder



F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0

Depinning transition

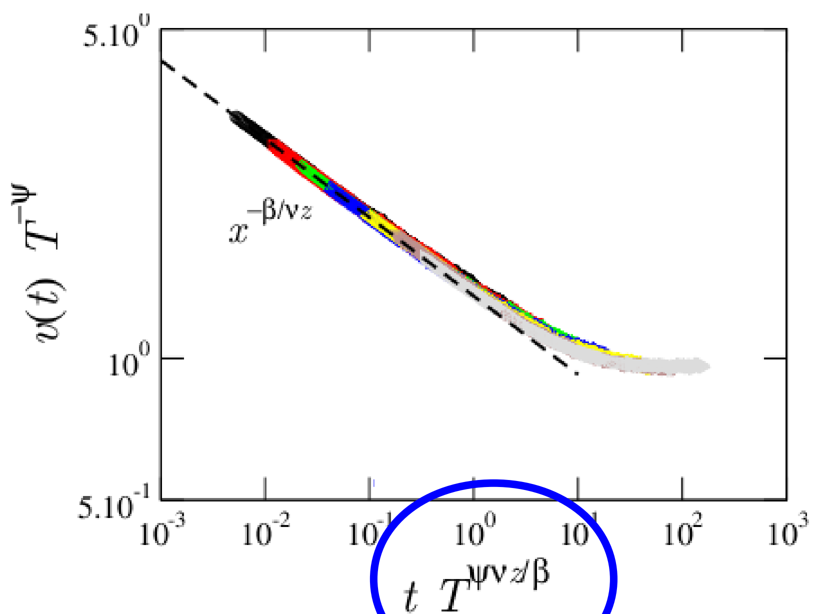
Thermal rounding

finite temperature depinning: **Thermal rounding**

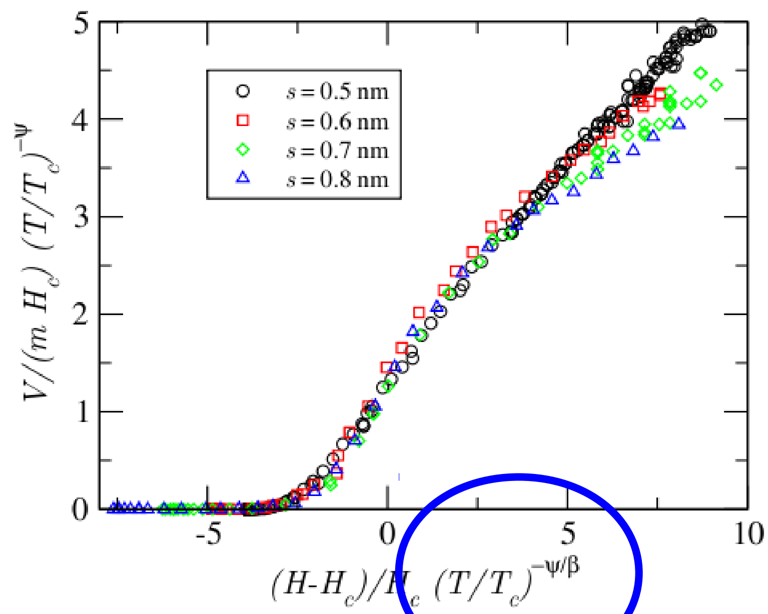
from standard critical phenomena

$$F = F_c \Rightarrow V \sim T^{\psi}$$

$$\psi = 0.15 \pm 0.01$$



Bustingorry, Kolton, Giamarchi, PRE 2012



Bustingorry, Kolton, Giamarchi, PRB 2012

Quenched disorder

Depinning transition

F	T	Δ
0	0	0
0	>0	0
0	0	>0
0	>0	>0
>0	0	0
>0	>0	0
>0	0	>0
>0	>0	>0